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COMPUTER PROGRAMS FOR CHARACTERISTIC MODES OF BODIES OF REVOLUTION

by

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Electrical Engineering Department
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Syracuse, New York 13210

Contract No. F19628-68-C-0180
Project No. 5635
Task No. 563506
Work Unit No. 56350601

SCIENTIFIC REPORT NO. 10

January 1971

Contract Monitor: John F. McIlvenna
Microwave Physics Laboratory

Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS 01730

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ABSTRACT

Computer programs are given for calculating the characteristic currents and characteristic gain patterns of conducting bodies of revolution. Also given are computer programs for using these characteristic currents in aperture radiation and plane-wave scattering problems. Plot programs for use with a Calcomp plotter are included. Operating procedures and program details are discussed, and sample input-output data are given.

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I. INTRODUCTION

The general theory and method of computation of characteristic modes for conducting bodies of arbitrary shape is given in reference [1]. The notation of this report is consistent with that of [1], which should be referred to for further identification of the symbols used. The programs given here are those used for the numerical results presented in reference [1].

Six computer programs have been written. They are defined according to their function:

1. Calculate the generalized impedance matrix Z
2. Calculate the characteristic currents (eigencurrents).
3. Plot the eigencurrents.
4. Calculate and plot the gain patterns of the eigencurrents
5. Calculate and plot σ/λ^2 (scattering cross section divided by the square of the wavelength) for an axially incident plane wave
6. Calculate and plot the gain pattern for radiation from an axially symmetric excitation.

These programs are discussed and listed in the next six sections Operating instructions and sample input-output data are also given.

II. GENERALIZED IMPEDANCE MATRIX

Program #1 calculates the elements of the generalized impedance matrix for a body of revolution, and is a slight modification of the computer program appearing in Appendix A of reference [2]. The subroutine LINEQ and its call statement 81 have been removed in order to obtain the generalized impedance matrix $\hat{Z}$ instead of the generalized admittance matrix Except for NPHI which was taken to be 20 in all of our work, all the punched card data required by program #1 is explained in reference [2]. As on page 26 of reference [2], punched card data is read early in the main program according to

```
50   READ(1,51,END = 52) NN, NP, NPHI, BK
51   FORMAT(3I3, E14.7)
     READ(1,53)(RH(I), I = 1, NP)
     READ(1,53)(ZH(I), I = 1, NP)
53   FORMAT(10F8.4)
```

Here, BK is the propagation constant $k = \omega\sqrt{\mu\epsilon}$. An odd number NP of data points are taken from the generating curve C of the body of revolution. RH(I) and ZH(I) are respectively the distance ρ from the axis of the body of revolution and the corresponding z coordinate at the I^{th} data point. The first and NP^{th} data points are on the ends of C. If C closes upon itself, care must be taken to make the coordinates of the first data point equal to those of the NP^{th}. Program #1 writes the NM2 by NM2 impedance matrix

$$\hat{Z}_n = \begin{bmatrix} \hat{Z}_n^{tt} & \hat{Z}_n^{t\phi} \\ \hat{Z}_n^{\phi t} & \hat{Z}_n^{\phi\phi} \end{bmatrix} \quad \text{where } n = NN \qquad (1)$$

on record 1 of direct access data set 6. NM2 is either NP-1 or NP-3 depending on whether or not C closes upon itself. If NN = 0, zeros are supplied for $\hat{Z}_o^{\phi t}$ and $\hat{Z}_o^{t\phi}$.

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$$\hat{Z}_n = \begin{bmatrix} \hat{Z}_n^{tt} & \hat{Z}_n^{t\phi} \\ \hat{Z}_n^{\phi t} & \hat{Z}_n^{\phi\phi} \end{bmatrix} \quad \text{where } n = NN \tag{1}$$

on record 1 of direct access data set 6. NM2 is either NP-1 or NP-3 depending on whether or not C closes upon itself. If NN = 0, zeros are supplied for $\hat{Z}_o^{\phi t}$ and $\hat{Z}_o^{t\phi}$.

```
//               (0034,EE,4,2),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX A3,A4,Z(1600),GS(40),G(5292),U
      DIMENSION RH(43),ZH(43),DH(43),TJ(20)
      DIMENSION SV(42),CV(42),ZS(42),R(42),ANG(40),AC(40),CSM(120)
      DIMENSION TP(80),T(80),TR(80),JK(4)
      REWIND 6
      U=(0.,1.)
   50 READ(1,51,END=52) NN,NP,NPHI,BK
   51 FORMAT(3I3,E14.7)
      READ(1,53)(RH(I),I=1,NP)
      READ(1,53)(ZH(I),I=1,NP)
   53 FORMAT(10F8.4)
   76 WRITE(3,54) NN,NP,NPHI,BK
   54 FORMAT(1X//' NN=',I3,' NP=',I3,' NPHI=',I3,' BK=',E14.7)
   55 FORMAT(1X/' RH')
      WRITE(3,55)
      WRITE(3,46)(RH(I),I=1,NP)
   46 FORMAT(1X,10F8.4)
      WRITE(3,56)
   56 FORMAT(1X/' ZH')
      WRITE(3,46)(ZH(I),I=1,NP)
      IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 58
      RH(NP+1)=RH(2)
      ZH(NP+1)=ZH(2)
      RH(NP+2)=RH(3)
      ZH(NP+2)=ZH(3)
      NP=NP+2
   58 DO 57 I=2,NP
      I2=I-1
      RR1=RH(I)-RH(I2)
      RR2=ZH(I)-ZH(I2)
      DH(I2)=SQRT(RR1*RR1+RR2*RR2)
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      R(I2)=.5*(RH(I)+RH(I2))
      SV(I2)=RR1/DH(I2)
      CV(I2)=RR2/DH(I2)
   57 CONTINUE
      KG=NP-1
      N=KG/2
      NM=N-1
      NM2=NM*2
      NM4=NM*4
      NZ=NM2*NM2
      NG=KG*KG
      M5=NN+2
      M6=NN+4
      FM=NN
      FM2=NN*NN
      PI=3.141593
      ETA=376.707
      DP=PI/NPHI
      CA=BK*BK*ETA
      CQ=ETA
      SS=0.
      DO 117 I=1,NM
      I1=2*(I-1)+1
      I2=I1+1
      SS=SS+DH(I1)+DH(I2)
```

```
COMPLEX Z(NZ)
DIMENSION RH(NP), ZH(NP), U(NZ), R(NZ), T2(NZ),
   A22(NZ), B(NZ), X(NZ), A(NZ), Y(NZ), T3(NZ),
FI(NZ), EU(NM2), RU(NM2), AMD(NM2), LB(NM2),
   MB(NM2)
```

where

```
NM2 = NP - 3
NZ = NM2 * NM2
```

The above allocations are based upon the value of NP after execution of statement 145

```
      DO 13 K=1,NPHI
      K2=K+M4
      G(M2)=G(M2)+GS(K)*CSM(K2)
   13 CONTINUE
   68 CONTINUE
   17 CONTINUE
   16 CONTINUE
      DO 74 J=1,NM
      J2=2*(J-1)+1
      J3=J2+1
      J4=J3+1
      J5=J4+1
      J6=4*(J-1)+1
      J7=J6+1
      J8=J7+1
      J9=J8+1
      DEL1=DH(J2)+DH(J3)
      DEL2=DH(J4)+DH(J5)
      TP(J6)=DH(J2)/DEL1
      TP(J7)=DH(J3)/DEL1
      TP(J8)=-DH(J4)/DEL2
      TP(J9)=-DH(J5)/DEL2
      T(J6)=DH(J2)*DH(J2)/2./DEL1
      T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
      T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
      T(J9)=DH(J5)*DH(J5)/2./DEL2
   74 CONTINUE
      DO 75 J=1,NM4
      TR(J)=T(J)
   75 CONTINUE
  115 IF((ZH(1)-ZH(NP-2)).EQ.0..AND.(RH(1)-RH(NP-2)).EQ.0.) GO TO 78
      IF(RH(1)) 77,23,77
   77 DEL1=DH(1)+DH(2)
      TR(1)=DH(1)*(1.+(DH(2)+DH(1)/2.)/DEL1)
      TR(2)=DH(2)*(1.+DH(2)/2./DEL1)
   23 IF(RH(NP)) 79,78,79
   79 J1=(N-2)*4+3
      J2=J1+1
      DEL2=DH(NP-2)+DH(KG)
      TR(J1)=DH(NP-2)*(1.+DH(NP-2)/2./DEL2)
  116 TR(J2)=DH(KG)*(1.+(DH(NP-2)+DH(KG)/2.)/DEL2)
   78 DO 30 J=1,NM
      JL=(J-1)*NM2
      J3=(J-1)*4
      J1=2*(J-1)
      DO 31 I=1,NM
      L1=JL+I
      L2=L1+NM
      L3=NM*NM2+L1
      L4=L3+NM
      Z(L1)=0.
      Z(L2)=0.
      Z(L3)=0.
      Z(L4)=0.
      I1=2*(I-1)
      I3=(I-1)*4
      DO 70 JJ=1,4
      J2=J1+JJ
      J7=J3+JJ
      DO 71 II=1,4
```

6

```
      I2=I1+II
      I7=I3+II
      J4=(J2-1)*KG+I2
      J5=J4+NG
      J6=J5+NG
      SS=SV(I2)*SV(J2)
      CC=CV(I2)*CV(J2)
      A3=.5*(G(J6)+G(J4))
      A4=.5*(G(J6)-G(J4))
      Z(L1)=Z(L1)+(CA*T(I7)*T(J7)*(SS*A3+CC*G(J5))-CO*TP(I7)*TP(J7)*G(J5
     1))*U
      Z(L2)=Z(L2)+CA*SV(J2)*TR(I7)*T(J7)*A4-FM*CO*G(J5)*TR(I7)*TP(J7)/R(
     1I2)
      Z(L3)=Z(L3)-CA*SV(I2)*T(I7)*TR(J7)*A4+FM*CO*G(J5)*TP(I7)*TR(J7)/R(
     1J2)
      Z(L4)=Z(L4)+(CA*A3-FM2*CO/R(I2)/R(J2)*G(J5))*TR(I7)*TR(J7)*U
   71 CONTINUE
   70 CONTINUE
   31 CONTINUE
   30 CONTINUE
      WRITE(6)(Z(I),I=1,NZ)
   88 FORMAT(1X,10G11.4)
      JK(1)=1
      JK(2)=N
      JK(3)=NM2*NM+1
      JK(4)=JK(3)+NM
      DO 93 J=1,4
      K1=JK(J)
      WRITE(3,24) J
   24 FORMAT(1X/' Z',I1)
      DO 92 I=1,NM
      K2=K1+NM-1
   96 WRITE(3,88)(Z(K),K=K1,K2)
      K1=K1+NM2
   92 CONTINUE
   93 CONTINUE
      GO TO 50
   52 STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                  X
//             VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
  1 21 20 0.3141593E+00
  0.0000  0.5000  1.0000  1.5000  2.0000  2.5000  3.0000  3.5000  4.0000  4.5000
  5.0000  5.5000  6.0000  6.5000  7.0000  7.5000  8.0000  8.5000  9.0000  9.5000
 10.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000
/*
//
```

VN= 1 NP= 21 NPHI= 2, RK= 0.3141593E 1

RH

0.0 0.5000 1.0000 1.5000 2.0000 2.5000 3.0000 3.5000 4.0000 4.5000
5.0000 5.5000 6.0000 6.5000 7.0000 7.5000 8.0000 8.5000 9.0000 9.5000
10.0000

ZH

0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0
0.0

TJ

1.0000 2.0000 3.0000 4.0000 5.0000 6.0000 7.0000 8.0000 9.0000

Z1

33.25	-3173.	25.91	834.4	35.07	299.0	32.63	107.5	29.78	53.31
26.47	28.44	22.90	14.47	19.17	5.839	15.46	0.2443		
36.75	817.6	36.72	-1031.	34.16	437.9	31.87	185.1	29.01	69.14
25.83	34.12	22.32	17.21	18.82	7.455	15.24	1.395		
35.03	302.9	34.07	430.4	32.63	-1090.	30.44	297.6	27.76	131.0
24.79	48.64	21.58	23.5	18.24	10.61	14.89	3.469		
32.71	109.1	31.73	186.3	30.37	293.5	28.48	-820.7	26.18	224.3
23.4	98.66	21.49	34.19	17.46	16.37	14.41	6.819		
29.8	52.49	29.91	63.43	27.74	132.0	26.13	221.6	24.01	-658.8
21.83	179.4	13.15	77.33	16.49	28.36	13.81	12.41		
26.48	26.52	25.84	34.21	24.79	48.79	23.38	99.34	21.79	177.5
19.64	-550.0	17.77	149.7	15.37	52.63	13.09	23.43		
22.93	14.51	22.46	17.75	21.59	23.12	20.49	36.30	19.14	77.87
17.74	148.4	15.72	-471.5	14.32	129.3	12.28	52.05		
19.19	5.831	19.93	7.478	18.25	10.54	17.46	16.41	16.49	28.43
15.36	62.00	14.31	128.3	12.49	-412.2	11.64	114.9		
15.47	0.2559	15.25	1.473	14.90	3.454	14.41	6.840	13.81	12.44
13.03	23.48	12.27	52.33	11.62	114.1	10.53	-365.8		

Z2

-1017.	37.45	434.4	33.42	87.31	31.37	34.93	27.24	24.87	21.81
22.72	15.39	22.02	9.853	21.02	4.029	36.83	-5.493		
-911.3	36.42	-275.4	34.33	163.4	30.74	48.25	26.47	27.41	21.20
23.17	15.47	21.87	9.616	20.64	3.969	36.03	-5.208		
-237.5	34.73	-119.4	32.64	-78.05	29.42	91.83	25.20	35.79	20.22
24.63	14.79	21.96	9.231	20.11	3.870	34.73	-4.748		
-96.12	32.42	-109.5	30.43	-157.4	27.42	-32.97	23.50	63.12	18.89
29.02	13.87	22.34	8.712	19.57	3.736	33.05	-4.139		
-49.53	29.43	-51.52	27.73	-59.55	25.31	-88.97	21.46	-13.40	17.28
47.83	12.73	25.41	6.577	19.46	3.557	31.33	-3.409		
-26.74	26.14	-27.15	24.65	-28.58	22.26	-34.07	19.14	-54.12	15.46
-4.247	11.45	37.77	7.339	21.13	3.358	30.03	-2.595		
-13.53	22.59	-13.75	21.31	-14.30	19.79	-15.71	16.62	-20.06	13.48
-34.93	10.76	3.847E-1	6.543	29.93	3.130	30.58	-1.736		
-5.375	18.30	-5.572	17.35	-5.773	16.19	-6.945	14.01	-8.577	11.43
-12.41	8.236	-24.13	5.714	1.323	2.837	38.27	-0.8873		
3.4993E-1	15.71	-.3263	14.4	-0.9573	13.10	-1.864	11.39	-3.122	9.363
-5.737	7.145	-.51	4.854	-12.93	2.633	24.55	-0.36510E-01		

Z3

1930.	-37.59	898.2	-35.33	228.1	-34.64	25.58	-32.27	49.33	-29.39
26.67	-26.10	13.54	-22.55	5.281	-18.97	-0.6111E-01	-15.19		
-432.9	-35.46	230.3	-34.37	317.7	-32.44	108.8	-31.40	51.31	-27.70
27.05	-24.62	13.73	-21.33	5.547	-17.33	0.2142	-14.38		
-87.92	-31.49	-162.7	-30.33	79.84	-29.45	156.0	-27.42	59.26	-24.49
28.43	-22.24	14.25	-19.26	6.047	-16.17	0.945	-13.18		
-34.93	-27.26	-48.43	-25.49	-91.83	-25.23	33.81	-23.52	88.75	-21.46
33.92	-19.12	15.65	-16.61	6.917	-13.99	1.355	-11.37		
-24.86	-21.33	-27.41	-21.22	-35.87	-20.24	-63.10	-18.92	13.86	-17.30
54.01	-15.46	19.08	-13.43	3.546	-11.42	3.105	-9.351		
-22.71	-15.92	-23.18	-15.49	-24.65	-14.81	-29.87	-13.88	-47.87	-12.75
4.322	-11.46	34.87	-10.[illegible]7	12.36	-8.652	4.983	-7.137		
-22.02	-9.873	-21.36	-9.635	-21.80	-9.248	-22.34	-8.727	-25.44	-8.091
-37.77	-7.351	0.9157E-01	-6.552	24.10	-5.717	8.437	-4.853		
-21.01	-4.047	-20.64	-3.936	-20.11	-3.886	-19.58	-3.749	-19.47	-3.579
-21.21	-3.378	-29.93	-3.142	-1.204	-2.834	18.07	-2.635		
-36.83	5.455	-36.[illegible]	5.182	-34.70	4.724	-33.[illegible]	4.116	-31.33	3.389
-30.03	2.577	-30.6[illegible]	1.722	-39.29	0.3746	-24.45	0.7413E-01		

Z4

37.42	-3342.	35.09	-333.6	31.60	-65.38	27.05	-29.52	21.74	-22.40
15.94	-21.29	10.01	-20.94	4.285	-20.19	-4.763	-35.68		
35.05	-338.1	33.08	-224.4	29.91	-51.90	25.75	-23.53	20.86	-18.00
15.53	-17.73	10.07	-18.55	4.790	-17.76	-2.781	-31.94		
31.58	-65.82	29.90	-62.24	27.19	-34.13	23.64	-10.68	19.45	-9.961
14.87	-11.81	10.15	-13.26	5.579	-13.78	0.3489	-25.89		
27.06	-29.59	25.74	-23.33	23.64	-10.68	20.86	7.629	17.58	5.183
13.98	-2.828	10.24	-6.524	6.581	-8.355	4.383	-17.75		
21.74	-22.42	20.86	-18.01	19.45	-9.947	17.58	5.240	15.36	21.00
12.85	12.55	10.29	2.679	7.702	-1.465	9.305	-7.729		
15.94	-21.21	15.53	-17.73	14.87	-11.80	13.98	-2.803	12.89	12.62
11.64	27.01	10.27	17.38	8.838	7.349	13.86	4.079		
10.01	-20.95	10.07	-18.05	10.16	-13.26	10.25	-6.522	10.30	2.704
10.28	17.43	10.15	31.47	9.880	20.76	18.56	18.19		
4.289	-20.20	4.794	-17.76	5.584	-13.78	6.586	-8.348	7.708	-1.453
8.343	7.372	9.883	20.81	10.73	32.27	22.74	37.63		
-4.757	-35.58	-2.775	-31.94	0.3568	-25.89	4.392	-17.74	9.016	-7.718
13.87	4.098	18.57	18.22	22.75	37.69	53.85	127.4		

III. EIGENCURRENTS

Program #2, which calculates the eigencurrents, accepts punched card data according to

```
      READ(1,7) NN, NP
7     FORMAT(2I3)
      READ(1,39)(RH(I), I = 1, NP)
      READ(1,39)(ZH(I), I = 1, NP)
39    FORMAT(10F8.4)
```

The variables NN, NP, RH, and ZH have the same meaning as in program #1. The generalized impedance matrix Z is read from the first record of direct access data set 6 according to

```
      REWIND 6
      READ(6)(Z(I), I = 1, NZ)
```

where NZ is either $(NP-1)^2$ or $(NP-3)^2$ depending on whether or not C closes upon itself. Program #2 writes the eigencurrents on record 2 of direct access data set 6.

The R and X in DO loop 11 are the matrices [R] and [X] appearing in (2-25) of [1] computed from $[Z_n]$ of (2-47) of [1]. The matrix [X] is stored columnwise, but the matrix [R] is stored according to the symmetric mode used in the IBM System/360 Scientific Subroutine Package [4]. Statement 130 invokes the eigenvalue and eigenvector subroutine EIGEN in the Scientific Subroutine Package. The subroutine EIGEN puts the eigenvalues μ of (2-26) of [1] ordered $\mu_1 \geq \mu_2 \geq \cdots$ in the diagonal positions of R. The eigenvectors of the matrix [R] will appear in U. It has been observed that the eigenvectors obtained from EIGEN are normalized so that U is orthogonal in accordance with (2-26) of [1]. DO loop 104 stores the μ_i in EU. DO loop 75 puts the matrix [XU] appearing in (2-30) of [1] in T2. DO loop 78 puts the matrix $[A] = [\tilde{U} X U]$ of (2-30) of [1] in A. Upon exit from DO loop 70, JM is the dimension of the submatrix $[\mu_{11}]$ in (2-28) of [1]. DO loop 73 puts the submatrix $[A_{22}]$ appearing in (2-30) of [1] in A_{22}. Since MINV is the matrix inversion subroutine from the

Scientific Subroutine Package [4], statement 128 inverts the matrix $[A_{22}]$. DO loop 81 puts $[A_{22}]^{-1}$ $[\tilde{A}_{12}]$ in T3. DO loop 84 puts $[A_{12}][A_{22}]^{-1}[\tilde{A}_{12}]$ in B, using the symmetric mode of storage. Statement 129 finds the eigenvalues and eigenvectors of the matrix [B] appearing in (2-36) of [1]. DO loop 107 puts the eigenvalues λ of (2-36) of [1] in AMD. DO loop 91 puts $[\mu_{11}^{-1/2}y]$ of (2-37) of [1] in T2. Upon exit from DO loop 93, the matrix

$$\begin{bmatrix} [\delta] \\ -[A_{22}^{-1} \ \tilde{A}_{12}] \end{bmatrix} [\mu_{11}^{-1/2}y]$$

will be in T2.

The index J of DO loop 96 indicates the J^{th} eigencurrent. DO loop 98 puts the [I] of (2-37) of [1] in FI. Because $f_i(t)$ of (2-42) of [1] is defined by (30) of [3], statements 143 and 144 have to divide the elements of [I] by ρ in order to obtain the sinusoidal components of the eigencurrents. DO loop 137 sets the largest of these sinusoidal components equal to unity. At the time FI is printed, the eigencurrent (current per unit length) at the$(2*J+1)^{th}$ data point is given by

$$\vec{u}_t \ FI(J) + \vec{u}_\phi \ FI(J+NM) \qquad NN = 0$$

$$\vec{u}_t \ FI(J) \cos n\phi + \vec{u}_\phi \ FI(J+NM) \sin n\phi \qquad NN = n \neq 0$$

When NN = n $\neq$ 0, the alternate eigencurrent

$$\vec{u}_t \ FI(J) \sin n\phi - \vec{u}_\phi \ FI(J + NM) \cos n\phi$$

is also possible.

If NP > 41, some dimension statements must be changed. Minimum allocations are given by

```
        COMPLEX Z(NZ)
        DIMENSION RH(NP), ZH(NP), U(NZ), R(NZ), T2(NZ),
           A22(NZ), B(NZ), X(NZ), A(NZ), Y(NZ), T3(NZ),
        FI(NZ), EU(NM2), RU(NM2), AMD(NM2), LB(NM2),
           MB(NM2)
where   NM2 = NP - 3
        NZ = NM2 * NM2
```

The above allocations are based upon the value of NP after execution of statement 145

Listing of Program #2

```
//                    (0034,EE,2,2),'MAUTZ,JOE',MSGLEVEL=1
// EXEC SSPCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      COMPLEX Z(1444),U1,U2,U3
      DIMENSION RH(41),ZH(41),U(1444),R(1444),T2(1444),A22(1444),B(1444)
      DIMENSION X(1444),A(1444),Y(1444),T3(1444),FI(1444),EU(38),RU(38)
      DIMENSION AMD(38),LB(38),MB(38)
      EQUIVALENCE (R(1),T2(1),A22(1),B(1)),(X(1),A(1),Y(1))
      EQUIVALENCE (T3(1),FI(1)),(EU(1),AMD(1))
      READ(1,7) NN,NP
    7 FORMAT(2I3)
      WRITE(3,3) NN,NP
    3 FORMAT('1 NN NP'/1X,2I3)
      READ(1,39)(RH(I),I=1,NP)
      READ(1,39)(ZH(I),I=1,NP)
   39 FORMAT(10F8.4)
      WRITE(3,40)(RH(I),I=1,NP)
   40 FORMAT('ORH'/(1X,10F8.4))
      WRITE(3,41)(ZH(I),I=1,NP)
   41 FORMAT('0ZH'/(1X,10F8.4))
  145 IF((RH(1)-RH(NP)).EQ.0..AND.(ZH(1)-ZH(NP)).EQ.0.) NP=NP+2
  146 NM2=NP-3
      NM=NM2/2
      NZ=NM2*NM2
      REWIND 6
      READ(6)(Z(I),I=1,NZ)
      U3=(0.,1.)
      S1=.25
      IF(NN.EQ.0) S1=.5
      U2=S1*U3
      J5=0
      DO 11 J=1,NM2
      J2=(J-1)*NM2
      DO 12 I=1,J
      J5=J5+1
      J3=J2+I
      J4=(I-1)*NM2+J
      IF(J.GT.NM.AND.I.LE.NM) GO TO 28
      U1=S1*(Z(J3)+Z(J4))
      GO TO 29
   28 U1=U2*(Z(J4)-Z(J3))
   29 R(J5)=U1
      X(J3)=AIMAG(U1)
      X(J4)=X(J3)
   12 CONTINUE
   11 CONTINUE
  130 CALL EIGEN(R,U,NM2,0)
      J1=0
      DO 104 J=1,NM2
      J1=J1+J
      EU(J)=R(J1)
      RU(J)=1./SQRT(ABS(EU(J)))
  104 CONTINUE
      WRITE(3,141)(EU(J),J=1,NM2)
  141 FORMAT('0EIGENVALUES OF THE MATRIX R'/(1X,7E11.4))
      DO 75 J=1,NM2
      J1=(J-1)*NM2
      DO 76 I=1,NM2
      J2=J1+I
      T2(J2)=0.
```

```
      J3=(I-1)*NM2
      DO 77 K=1,NM2
      K1=K+J3
      K2=K+J1
      T2(J2)=T2(J2)+X(K1)*U(K2)
   77 CONTINUE
   76 CONTINUE
   75 CONTINUE
      DO 78 J=1,NM2
      J1=(J-1)*NM2
      DO 79 I=1,J
      J2=J1+I
      A(J2)=0.
      J3=(I-1)*NM2
      DO 80 K=1,NM2
      K1=K+J3
      K2=K+J1
      A(J2)=A(J2)+U(K1)*T2(K2)
   80 CONTINUE
      J4=J3+J
      A(J4)=A(J2)
   79 CONTINUE
   78 CONTINUE
      X2=EU(1)*1.E-03
      DO 70 J=1,NM2
      JM=J-1
      IF(EU(J).LT.X2) GO TO 72
   70 CONTINUE
   72 JN=NM2-JM
      JM1=JM+1
      J1=0
      DO 73 J=JM1,NM2
      J2=(J-1)*NM2
      DO 74 I=JM1,NM2
      J1=J1+1
      J3=J2+I
      A22(J1)=A(J3)
   74 CONTINUE
   73 CONTINUE
  128 CALL MINV(A22,JN,D,LB,MB)
      J1=0
      DO 81 J=1,JM
      J3=(J-1)*NM2+JM
      DO 82 I=1,JN
      J2=(I-1)*JN
      J1=J1+1
      T3(J1)=0.
      DO 83 K=1,JN
      K1=J2+K
      K2=J3+K
      T3(J1)=T3(J1)+A22(K1)*A(K2)
   83 CONTINUE
   82 CONTINUE
   81 CONTINUE
      J2=0
      DO 84 J=1,JM
      J3=(J-1)*NM2
      J5=(J-1)*JN
      DO 85 I=1,J
      J2=J2+1
```

```
      J4=J3+I
      B(J2)=A(J4)
      J6=(I-1)*NM2+JM
      DO 86 K=1,JM
      K1=K+J6
      K2=K+J5
      B(J2)=B(J2)-A(K1)*T3(K2)
   86 CONTINUE
      B(J2)=B(J2)*RU(J)*RU(I)
   85 CONTINUE
   84 CONTINUE
  129 CALL EIGEN(B,Y,JM,0)
      J1=0
      DO 107 J=1,JM
      J1=J1+J
      AMD(J)=B(J1)
  107 CONTINUE
      WRITE(3,58)(AMD(J),J=1,JM)
   58 FORMAT('OEIGENVALUES OF THE MATRIX B'/(1X,5E14.7))
      DO 91 J=1,JM
      J1=(J-1)*JM
      J4=(J-1)*NM2
      DO 92 I=1,JM
      J3=I+J4
      J2=I+J1
      T2(J3)=Y(J2)*RU(I)
   92 CONTINUE
   91 CONTINUE
      S1=0.
      DO 93 J=1,JM
      J1=(J-1)*NM2
      DO 94 I=1,JN
      J2=J1+I+JM
      T2(J2)=0.
      DO 95 K=1,JM
      K1=(K-1)*JN+I
      K2=K+J1
      T2(J2)=T2(J2)-T3(K1)*T2(K2)
   95 CONTINUE
   94 CONTINUE
   93 CONTINUE
      DO 96 J=1,JM
      S1=0.
      J1=(J-1)*NM2
      DO 97 I=1,NM
      J2=J1+I
      J3=J2+NM
      J4=2*I+1
      FI(J2)=0.
      FI(J3)=0.
      DO 98 K=1,NM2
      K2=K+J1
      K1=(K-1)*NM2+I
      K3=K1+NM
      FI(J2)=FI(J2)+U(K1)*T2(K2)
      FI(J3)=FI(J3)+U(K3)*T2(K2)
   98 CONTINUE
  143 FI(J2)=FI(J2)/RH(J4)
  144 FI(J3)=FI(J3)/RH(J4)
      S2=ABS(FI(J2))
```

```
      S3=ABS(FI(J3))
      IF(S2-S1) 136,133,133
  133 S1=S2
      J5=J2
  136 IF(S3-S1) 97,135,135
  135 S1=S3
      J5=J3
   97 CONTINUE
      S1=1./FI(J5)
      J2=J1+1
      J3=J1+NM
      J4=J3+1
      J5=J1+NM2
      WRITE(3,138) AMD(J)
  138 FORMAT('OLAMBDA = ',E11.4)
      DO 137 I=J2,J5
      FI(I)=FI(I)*S1
  137 CONTINUE
      WRITE(3,60)(FI(I),I=J2,J3)
   60 FORMAT(' JT',10F8.4/(3X,10F8.4))
      WRITE(3,61)
   61 FORMAT(' JO')
      WRITE(3,62)(FI(I),I=J4,J5)
   62 FORMAT('+ /',10F8.4/(3X,10F8.4))
   96 CONTINUE
      WRITE(6)(FI(I),I=1,J5)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                    X
//               VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
  1 21
  0.0000  0.5000  1.0000  1.5000  2.0000  2.5000  3.0000  3.5000  4.0000  4.5000
  5.0000  5.5000  6.0000  6.5000  7.0000  7.5000  8.0000  8.5000  9.0000  9.5000
 10.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000
/*
//
```

Output of Program #2

```
NN NP
 1 21

RH
 0.0     0.5000  1.0000  1.5000  2.0000  2.5000  3.0000  3.5000  4.0000  4.5000
 5.0000  5.5000  6.0000  6.5000  7.0000  7.5000  [illegible]  8.5000  9.0000  9.5000
 10.0000

ZH
 0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
 0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0     0.0
 0.0

EIGENVALUES OF THE MATRIX R
 0.1753E 03 0.3901E 02  .4532E 01 0.1546E 01  .1460E [illegible] 0.[illegible]E-01  .7320E-01
 0.4387E-01 0.3325E-02  .1777E-02 0.1043E-02  .1[illegible]E-03-0.2177E-02-0.3226E-02
-0.4590E-02-0.6731E-01-0.1598E 00-0.3073E 00

EIGENVALUES OF THE MATRIX B
 0.2559653E 01-0.8735475E-02-0.2644487E 02

LAMBDA =  0.2560E 01
JT  0.2460  0.2443  0.2234  0.1[illegible]  0.1685  0.1385  0.1003  0.9777  0.9353
J0 -0.2470 -0.2449 -0.1517 -0.1[illegible]  0.0064  0.0272  0.2390  0.1371  1.0000

LAMBDA = -0.3785E-02
JT -0.9777 -0.9503 -0.8871 -0.7994 -0.6966 -0.5803 -0.4374 -0.3293 -0.2532
J0  0.9853  1.0000  0.9210  0.3912  0.8220  0.7562  0.6965  0.6163  0.6348

LAMBDA = -0.2544E 02
JT -0.9716 -0.7251 -0.4462 -0.1293  0.2206  0.5424  0.8121  0.9287  0.8733
J0  1.0000  0.7113  0.8536  0.6437  0.5762  0.4611  0.2459  0.2519 -0.329[illegible]
```

IV. EIGENCURRENT PLOTS

Program #3 which plots the eigencurrents accepts punched card data according to

```
      READ(1,90) NM2, JM
90    FORMAT(20I3)
```

In the previous program an NM2 by NM2 generalized impedance matrix has led to JM eigencurrents. More precisely, NM2 is defined by statement 146 of the previous program and JM is the number of numbers printed under the heading "Eigenvalues of the matrix B" in the output of the previous program. The eigencurrents are read from the second record of direct access data set 6 according to

```
REWIND 6
READ(6)
NZ1 = NM2 * JM
READ(6)(FI(I), I = 1, NZ1)
```

DO loop 93 prepares the vertical coordinate for plotting and DO loop 95 prepares the horizontal coordinate. The origin is at (1.,5.). The horizontal axis corresponds to the contour length variable. If the NP data points defining the contour C are not equally spaced, this correspondence may become nonlinear because DO loop 95 always supplies equally spaced horizontal coordinates. The index J of DO loop 94 indicates the J^{th} eigencurrent. Statements 18 and 19 plot the sinusoidal components of J_t and J_ϕ respectively.

Minimum allocations for FI and X are given by

```
DIMENSION FI(NM2 * NM2), X(NM2/2).
```

Listing of Program #3

```
//               (0034,EE,2,2,,8),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      DIMENSION FI(1444),XP(4),YP(4),AREA(400),X(19)
      CALL PLOTS(AREA,400)
      READ(1,90) NM2,JM
   90 FORMAT(20I3)
      WRITE(3,91) NM2,JM
   91 FORMAT('0NM2 JM'/1X,2I3)
      NZ1=NM2 JM
      REWIND 6
      READ(6)
      READ(6)(FI(I),I=1,NZ1)
      NM=NM2/2
      XP(1)=1.
      XP(2)=6.
      YP(1)=5.
      YP(2)=5.
      XP(3)=1.
      XP(4)=1.
      YP(3)=3.
      YP(4)=7.
      DO 93 I=1,NZ1
      FI(I)=5.+FI(I)
   93 CONTINUE
      S1=5./(NM+1)
      DO 95 J=1,NM
      X(J)=1.+J*S1
   95 CONTINUE
      DO 94 J=1,JM
      J1=(J-1)*NM2+1
      J2=J1+NM
      CALL LINE(XP(1),YP(1),2,1,0,0)
      DO 96 K=2,6
      S1=8-K
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
   96 CONTINUE
      CALL LINE(XP(3),YP(3),2,1,0,0)
      DO 97 K=1,5
      S1=8-K
      CALL SYMBOL(1.,S1,.14,13,90.,-1)
   97 CONTINUE
      CALL NUMBER(.76,5.93,.14,1.,0.,-1)
      CALL NUMBER(.76,4.93,.14,0.,0.,-1)
      CALL NUMBER(.64,3.93,.14,-1.,0.,-1)
   18 CALL LINE(X(1),FI(J1),NM,1,4,1)
   19 CALL LINE(X(1),FI(J2),NM,1,0,1)
      CALL PLOT(7.,0.,-3)
   94 CONTINUE
      CALL PLOT(5.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                  X
//             VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
 13  3
/*
//
```

V. GAIN EIGENPATTERNS

Program #4 which calculates and plots the gain patterns of the eigencurrents accepts punched card data according to

```
      READ(1,10) NN, NP, NT, NS, JM, BK, SCL
10    FORMAT (5I3, 2E14.7)
      READ(1,11)(AMD(I), I = 1, JM)
11    FORMAT (5E14.7)
      READ(1,15)(RH(I), I = 1, NP)
      READ(1,15)(ZH(I), I = 1, NP)
15    FORMAT (10F8.4)
```

Here, NN, NP, BK, RH, and ZH are the same as in program #1 and JM is the same as in program #3. The electric field and gain will be computed at polar angles $\theta_i = \frac{(i-1)\pi}{NT-1}$, i=1,2,... NT but will be printed only at i=1, NS + 1, 2*NS + 1,... . One inch will correspond to a gain of 1./SCL on the plot. The variable AMD appears under the heading "Eigenvalues of the Matrix B" in the output of program #2. The JM eigencurrents are read from record 2 of direct access data set 6 according to

```
      REWIND 6
      READ(6)
      READ(6)(FI(J), J = 1, NZ1)
```

where NZ1 = NM2 * JM. The variable NM2 is the same as that appearing in the first three programs.

In DO loop 40, DH(I), RS(I), ZS(I), SV(I), and CV(I) are respectively the distance between the I^{th} and $(I+1)^{th}$ data point, and ρ, z, sin v, and cos v midway between the I^{th} and $(I+1)^{th}$ data points. Here, v is the angle between $\vec{u}_t$ and the z axis. In DO loop 41, T(4*(J-1) + I), I=1,2,3,4, is the amplitude of $\frac{T_J(t)}{\rho}$ (the J^{th} triangle function divided by the cylindrical coordinate ρ) multiplied by DH(2*(J-1)+I) and evaluated midway between the $(2*(J-1)+I)^{th}$ and $(2*(J-1)+I+1)^{th}$ data points. Now T represents the functions $f_i^t(t)$ associated with J_t and TR those functions $f_i^\phi(t)$ associated with

J_ϕ although only $f_i(t)$ appears in (2-42) of [1], no distinction being made there between $f_i^t(t)$ and $f_i^\phi(t)$. If the surface S has no edges, $f_i^t(t) = f_i^\phi(t)$, but if S has an edge at either end of C, program #4 modifies the $f_i^\phi(t)$ nearest this edge so that $f_i^\phi = \frac{2}{\rho}$ at the edge in an attempt to account for the singularity of J_ϕ at the edge.

The subroutine PLANE is concerned with the integrals

$$\mathcal{R}_n = \oiint_S \vec{W}_i \cdot \vec{u}_m \, e^{-j\vec{k}_m \cdot \vec{r}} \, ds \tag{2}$$

appearing in (2-51) of [1]. The $\mathcal{R}$ integrals can be written as

$$\mathcal{R}_n = \begin{bmatrix} \mathcal{R}_n^{t\theta} & \mathcal{R}_n^{\phi\theta} \\ \mathcal{R}_n^{t\phi} & \mathcal{R}_n^{\phi\phi} \end{bmatrix} \tag{3}$$

The row submatrices on the right hand side of (3) are defined according to which $\vec{W}_i$ and which $\vec{u}_m$ are used

Matrix element	$\vec{u}_m$	$\vec{W}_i$
$\mathcal{R}_n^{t\theta}$	$\vec{u}_\theta$	$\vec{u}_t f_i(t) \cos n\phi$
$\mathcal{R}_n^{\phi\theta}$	$\vec{u}_\theta$	$\vec{u}_\phi f_i(t) \sin n\phi$
$\mathcal{R}_n^{t\phi}$	$\vec{u}_\phi$	$\vec{u}_t f_i(t) \cos n\phi$
$\mathcal{R}_n^{\phi\phi}$	$\vec{u}_\phi$	$\vec{u}_\phi f_i(t) \sin n\phi$

If the coefficients I_i appearing in (2-51) of [1] are partitioned into column vectors I^t and I^ϕ corresponding to the $\vec{u}_t$ and $\vec{u}_\phi$ directed $\vec{W}_i$, the radiation field will be given by

$$\begin{bmatrix} E_\theta \\ \\ E_\phi \end{bmatrix} = \frac{-j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} \mathcal{R}_n^{t\theta} & \mathcal{R}_n^{\phi\theta} \\ \\ \mathcal{R}_n^{t\phi} & \mathcal{R}_n^{\phi\phi} \end{bmatrix} \begin{bmatrix} [I^t] \\ \\ [I^\phi] \end{bmatrix} \tag{4}$$

Furthermore, it will be shown that

$$\begin{bmatrix} E_\theta \\ \\ E_\phi \end{bmatrix} = \frac{-j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} \cos n\phi & 0 \\ \\ 0 & \sin n\phi \end{bmatrix} \begin{bmatrix} \hat{R}_n^{t\theta} & -j\hat{R}_n^{\phi\theta} \\ \\ j\hat{R}_n^{t\phi} & \hat{R}_n^{\phi\phi} \end{bmatrix} \begin{bmatrix} [I^t] \\ \\ [I^\phi] \end{bmatrix} \tag{5}$$

where the careted submatrices R_n are given by equations (77) and (81) of reference [3]. If the current has the polarization (2-43) of [1] instead of (2-42) of [1], the matrix $\begin{bmatrix} \cos n\phi & 0 \\ 0 & \sin n\phi \end{bmatrix}$ of (5) must be replaced by $\begin{bmatrix} \sin n\phi & 0 \\ 0 & -\cos n\phi \end{bmatrix}$. The submatrices $\mathcal{R}_n$ use sinusoidal $\vec{W}_i$ and depend upon the measurement azimuthal angle ϕ_m while the $\hat{R}_n$ use exponential functions and are evaluated at $\phi_m = 0$. Consider the contribution to E_θ from I_i^ϕ. According to (4),

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi \int \rho \, dt \int_0^{2\pi} d\phi \, \vec{u}_\phi(\phi) f_i(t) \sin n\phi \cdot \vec{u}_\theta(\phi_m) \psi(\phi-\phi_m) \tag{6}$$

where

$$\psi(\phi-\phi_m) = e^{-j\vec{k}_m \cdot \vec{r}} \tag{7}$$

Since the integrand of (6) is periodic in ϕ with period 2π, ϕ_m can be added to ϕ without changing the value of the integral.

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi \int \rho \, f_i(t) dt \int_0^{2\pi} d\phi \, \vec{u}_\phi(\phi+\phi_m) \cdot \vec{u}_\theta(\phi_m) \, \psi(\phi) \, \sin n(\phi+\phi_m) \tag{8}$$

But

$$\vec{u}_\phi(\phi+\phi_m) \cdot \vec{u}_\theta(\phi_m) = \vec{u}_\phi(\phi) \cdot \vec{u}_\theta(0) \tag{9}$$

so that

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi \int \rho\ f_i(t)\ dt \int_0^{2\pi} d\phi\ \vec{u}_\phi(\phi)\ \sin n(\phi+\phi_m) \cdot \vec{u}_\theta(0)\ \psi(\phi) \tag{10}$$

Now $\vec{u}_\theta(0)$ is the measurement plane wave coming from the direction $\phi = 0$. Alternatively, (10) can be obtained directly from (6) by reasoning that (6) should depend only on the phase difference between the current $\vec{J}$ and the direction ϕ_m from which the measurement plane wave comes. For instance, one should be able to turn both $\vec{J}$ and ϕ_m back by ϕ_m to obtain (10). Equation (10) leads to

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi \ \frac{\hat{R}_n^{\phi\theta} - \hat{R}_{-n}^{\phi\theta}}{2j} \cos n\phi_m + \frac{\hat{R}_n^{\phi\theta} + \hat{R}_{-n}^{\phi\theta}}{2} \sin n\phi_m \tag{11}$$

Using the fact that $\hat{R}_n^{\phi\theta}$ is odd in n,

$$E_\theta = \frac{-j\omega\mu e^{-jkr}}{4\pi r} I_i^\phi\ (-j\ \hat{R}_n^{\phi\theta} \cos n\phi_m) \tag{12}$$

in agreement with (5). The rest of (5) can be similarly verified.

The subroutine PLANE is essentially the same as the one appearing in Appendix B of reference [2]. For the L^{th} measurement polar angle θ = THR(L) and the J^{th} function $f_J(t)$ of (2-42), PLANE stores $\hat{R}_n^{t\theta}$, $-j\hat{R}_n^{\phi\theta}$, $j\hat{R}_n^{t\phi}$, and $\hat{R}_n^{\phi\phi}$ in VVR(L1+J), VVR(L1+J+NM), VVR(L1+J+NM*2), and VVR(L1+J+NM*3) respectively where L1 = (L-1)*NM*4 and NM = NM2/2.

DO loop 92 multiplies the eigencurrents by ρ to retrieve

$$[I] = \begin{bmatrix} [I^t] \\ \\ [I^\phi] \end{bmatrix} \tag{13}$$

appearing in (2-25) of [1]. The i^{th} elements of $[I^t]$ and $[I^\phi]$ are the coefficients of the expansion functions $\vec{u}_t f_i(t) \cos n\phi$ and $\vec{u}_\phi f_i(t) \sin n\phi$ respectively for an eigencurrent of polarization (2-42) of [1].

The index KK of DO loop 81 indicates the KKth eigencurrent. In DO loop 83, K=1 obtains E_θ and K=2 obtains E_ϕ. DO loop 82 obtains the NT polar angles θ. Inner DO loop 84 performs the actual matrix multiplication indicated by (5). The gains G_θ and G_ϕ are proportional to $|E_\theta|^2$ and $|E_\phi|^2$. The average of the total gain over the area of the radiation sphere is unity.

$$\int_0^\pi (G_\theta(\theta) + G_\phi(\theta)) \sin \theta d\theta = \begin{cases} 2 & NN = 0 \\ 4 & NN \geq 1 \end{cases} \tag{14}$$

DO loop 85 stores E_θ and G_θ in positions 1 to NT of E and G and E_ϕ and G_ϕ in positions NT+1 to 2*NT of E and G. The phase of E is normalized to $-je^{-jkr}$. The magnitude of E is normalized so that $|E_\theta|^2 = G_\theta$ and $|E_\phi|^2 = G_\phi$. Statements 34 to 86 are concerned with plotting the gains of the eigencurrents.

Minimum allocations are given by

```
COMPLEX VR(NT*NM2*2), E(NT*2)
COMMON RS(NP-1),ZS(NP-1), SV(NP-1), CV(NP-1), T(NM2*2), TR(NM2*2)
DIMENSION AMD(JM), RH(NP), ZH(NP), DH(NP-1), TH(NT), G(NT*2), SN(NT*2)
      CS(NT*2), FI(NM2*NM2), GX(NT*2), GY(NT*2)
DIMENSION BJ(3*(NP-1))
```

Here, NP is the value of NP after execution of statement 96 in the main program. Also, BJ appears in PLANE.

Listing of Program #4

```
//               (0034,EE,3,2,,5),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE PLANE(VVR,THR,NT)
      COMPLEX VVR(1),A5,A6,U
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      DIMENSION BJ(126),THR(1),FK(20)
      KG=NP-1
      NM=KG/2-1
      M2=NN+2
      A5=2.*3.141593*U**(NN+1)
      NV=NM*4
      FK(1)=1.
      DO 153 J=1,M2
      J1=J+1
      FK(J1)=FK(J)*J
  153 CONTINUE
      DO 156 L=1,NT
      L1=(L-1)*NV
      CS=COS(THR(L))
      SN=SIN(THR(L))
      BCS=BK*CS
      DO 302 J=1,KG
      X=RS(J)*BK*SN
      J1=J
      I1=NN
      IF(I1) 303,304,303
  304 I1=I1+1
      J1=J1+KG
  303 DO 305 JJ=I1,M2
      IF(X-1.E-5) 1,1,2
    1 IF(JJ-1) 3,3,4
    3 BJ(J1)=1.
      GO TO 306
    4 BJ(J1)=0.
      GO TO 306
    2 RH=X/2.
      RH2=RH*RH
      RH3=RH**(JJ-1)
      BJ(J1)=RH3/FK(JJ)
      SS=BJ(J1)
    8 SST=SS*1.E-7
      DO 155 K=1,20
      SS=-SS*RH2/K/(K+JJ-1)
      BJ(J1)=BJ(J1)+SS
      IF(ABS(SS)-SST) 306,306,155
  155 CONTINUE
      STOP 155
  306 J1=J1+KG
  305 CONTINUE
  302 CONTINUE
      IF(NN) 307,308,307
  308 DO 309 J=1,KG
      J1=J+2*KG
      BJ(J)=-BJ(J1)
  309 CONTINUE
  307 DO 300 J=1,NM
      J1=J+L1
      J2=J1+NM
      J3=J2+NM
```

```
      J4=J3+NM
      VVR(J1)=0.
      VVR(J2)=0.
      VVR(J3)=0.
      VVR(J4)=0.
      DO 301 I=1,4
      I1=2*(J-1)+I
      I4=4*(J-1)+I
      I2=I1+KG
      I3=I2+KG
      A6=(COS(ZS(I1)*BCS)+U*SIN(ZS(I1)*BCS))*A5
      BJ1=(BJ(I3)+BJ(I1))*.5
      BJ2=(BJ(I3)-BJ(I1))*.5
      VVR(J1)=VVR(J1)+A6*(CS*SV(I1)*BJ2+SN*CV(I1)*BJ(I2)*U)*T(I4)
      VVR(J2)=VVR(J2)+A6*CS*BJ1*TR(I4)
      VVR(J3)=VVR(J3)+A6*SV(I1)*BJ1*T(I4)
      VVR(J4)=VVR(J4)+A6*BJ2*TR(I4)
  301 CONTINUE
  300 CONTINUE
  156 CONTINUE
      RETURN
      END
      COMPLEX U,U1,VR(5548),E(146)
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      DIMENSION AMD(38),RH(41),ZH(41),DH(40),TH(73),XP(2),YP(2),G(146)
      DIMENSION SN(73),CS(73),FI(1444),GX(146),GY(146),AREA(400)
      CALL PLOTS(AREA,400)
      READ(1,10) NN,NP,NT,NS,JM,BK,SCL
   10 FORMAT(5I3,2E14.7)
      READ(1,11)(AMD(I),I=1,JM)
   11 FORMAT(5E14.7)
      READ(1,15)(RH(I),I=1,NP)
      READ(1,15)(ZH(I),I=1,NP)
   15 FORMAT(10F8.4)
      WRITE(3,33)
   33 FORMAT('1 NN NP NT NS JM',6X,'BK',12X,'SCL')
      WRITE(3,12) NN,NP,NT,NS,JM,BK,SCL
   12 FORMAT(1X,5I3,2E14.7)
      WRITE(3,5)(AMD(I),I=1,JM)
    5 FORMAT('0AMD'/(1X,5E14.7))
      WRITE(3,16)(RH(I),I=1,NP)
   16 FORMAT('0RH'/(1X,10F8.4))
      WRITE(3,18)(ZH(I),I=1,NP)
   18 FORMAT('0ZH'/(1X,10F8.4))
      U=(0.,1.)
      PI=3.141593
      KL=1
      IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 96
      KL=0
      RH(NP+1)=RH(2)
      ZH(NP+1)=ZH(2)
      RH(NP+2)=RH(3)
      ZH(NP+2)=ZH(3)
      NP=NP+2
   96 NM2=NP-3
      NM=NM2/2
      NM4=NM*4
      NT2=NT*2
      NZ1=NM2*JM
      REWIND 6
```

```
      READ(6)
      READ(6)(FI(J),J=1,NZ1)
      DO 40 I=2,NP
      I2=I-1
      RR1=RH(I)-RH(I2)
      RR2=ZH(I)-ZH(I2)
      DH(I2)=SQRT(RR1*RR1+RR2*RR2)
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      SV(I2)=RR1/DH(I2)
      CV(I2)=RR2/DH(I2)
   40 CONTINUE
      DO 41 J=1,NM
      J2=2*(J-1)+1
      J3=J2+1
      J4=J3+1
      J5=J4+1
      J6=4*(J-1)+1
      J7=J6+1
      J8=J7+1
      J9=J8+1
      DEL1=DH(J2)+DH(J3)
      DEL2=DH(J4)+DH(J5)
      T(J6)=DH(J2)*DH(J2)/2./DEL1
      T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
      T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
      T(J9)=DH(J5)*DH(J5)/2./DEL2
   41 CONTINUE
      DO 97 J=1,NM4
      TR(J)=T(J)
   97 CONTINUE
      IF(KL.EQ.0) GO TO 98
      IF(RH(1)) 23,24,23
   23 DEL1=DH(1)+DH(2)
      TR(1)=DH(1)*(1.+(DH(2)+DH(1)/2.)/DEL1)
      TR(2)=DH(2)*(1.+DH(2)/2./DEL1)
   24 IF(RH(NP)) 26,98,26
   26 J1=(NM-1)*4+3
      J2=J1+1
      DEL2=DH(NP-2)+DH(NP-1)
      TR(J1)=DH(NP-2)*(1.+DH(NP-2)/2./DEL2)
      TR(J2)=DH(NP-1)*(1.+(DH(NP-2)+DH(NP-1)/2.)/DEL2)
   98 DEL=PI/(NT-1)
      C2=SCL*4./DEL
      IF(NN.EQ.0) C2=C2*.5
      DO 43 J=1,NT
      TH(J)=(J-1)*DEL
      SN(J)=SCL*SIN(TH(J))
      CS(J)=SCL*COS(TH(J))
   43 CONTINUE
      XP(1)=2.
      XP(2)=8.
      YP(1)=5.
      YP(2)=5.
      CALL PLANE(VR,TH,NT)
      C1=180./PI
      DO 31 J=1,NT
      TH(J)=TH(J)*C1
   31 CONTINUE
      DO 92 J=1,JM
```

```
      J1=(J-1)*NM2
      DO 93 I=1,NM
      J2=J1+I
      J3=J2+NM
      J4=2*I+1
      FI(J2)=FI(J2)*RH(J4)
      FI(J3)=FI(J3)*RH(J4)
   93 CONTINUE
   92 CONTINUE
      DO 81 KK=1,JM
      K1=(KK-1)*NM2
      S2=0.
      DO 83 K=1,2
      K2=(K-1)*NT
      K3=(K-1)*NM2
      DO 82 J=1,NT
      J1=(J-1)*NM4+K3
      U1=0.
      DO 84 I=1,NM2
      J3=I+J1
      J4=I+K1
      U1=U1+VR(J3)*FI(J4)
   84 CONTINUE
      J5=J+K2
      E(J5)=U1
      S1=CABS(U1)
      G(J5)=S1*S1
      S2=S2+G(J5)*SN(J)
   82 CONTINUE
   83 CONTINUE
      S3=SQRT(C2/S2)
      DO 85 J=1,NT2
      E(J)=E(J)*S3
      S2=CABS(E(J))
      G(J)=S2*S2
   85 CONTINUE
   34 CALL LINE(XP,YP,2,1,0,0)
      DO 77 J=1,7
      S1=9-J
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
   77 CONTINUE
      CALL LINE(YP,XP,2,1,0,0)
      DO 78 J=1,7
      S1=9-J
      CALL SYMBOL(5.,S1,.14,13,90.,-1)
   78 CONTINUE
      DO 86 K=1,2
      K2=(K-1)*NT
      DO 87 J=1,NT
      J1=J+K2
      S1=G(J1)*SN(J)
      GX(J)=5.+S1
      GY(J)=5.+G(J1)*CS(J)
      J2=NT2-J+1
      GX(J2)=5.-S1
      GY(J2)=GY(J)
   87 CONTINUE
      CALL LINE(GX,GY,NT2,1,0,0)
   86 CONTINUE
      WRITE(3,27) AMD(KK)
```

```
   27 FORMAT('0ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA =',E11
     1.4)
      WRITE(3,28)
   28 FORMAT('0      0     REAL(EO)     IMAG(EO)       GAINO      REAL(EO)     I
     1MAG(EO)      GAINO')
      WRITE(3,29)
   29 FORMAT('+     -',10X,'-',11X,'-',11X,'-',11X,'/',11X,'/',11X,'/')
      DO 30 K=1,NT,NS
      K2=NT+K
      WRITE(3,32) TH(K),F(K),G(K),F(K2),G(K2)
   32 FORMAT(1X,F6.1,6E12.4)
   30 CONTINUE
      CALL PLOT(7.,0.,-3)
   81 CONTINUE
      CALL PLOT(6.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                    X
//               VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
  1 21 73  4  3 0.3141593E+00 0.5000000E+00
 0.2559665E+01-0.8785684E-02-0.2644527E+02
  0.0000  0.5000  1.0000  1.5000  2.0000  2.5000  3.0000  3.5000  4.0000  4.5000
  5.0000  5.5000  6.0000  6.5000  7.0000  7.5000  8.0000  8.5000  9.0000  9.5000
 10.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000
/*
//
```

NN	NP	NT	NS	JM	BK	SCL
1	21	73	4	3	0.3141593E 00	.5000000E 00

AMD

0.2559665E 01-0.4785684E-02-0.2644527E 02

RH

0.0	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000
5.0000	5.5000	6.0000	6.5000	7.0000	7.5000	8.0000	8.5000	9.0000	9.5000
10.0000									

ZH

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0									

ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA = 0.2559E 01

θ	REAL(Eθ)	IMAG(Eθ)	GAINθ	REAL(EØ)	IMAG(EØ)	GAINØ
0.0	-0.1304E 01	0.0	0.1699E 01	0.1304E 01	0.0	0.1699E 01
10.0	-0.1243E 01	0.0	0.1546E 01	0.1134E 01	0.0	0.1287E 01
20.0	-0.1080E 01	0.0	0.1165E 01	0.6803E 00	0.0	0.4628E [illegible]
30.0	-0.8558E 00	0.0	0.7324E 00	0.7477E-01	0.0	0.5591E-02
40.0	-0.6218E 00	0.0	0.3867E 00	-0.5331E 00	0.0	0.2842E [illegible]
50.0	-0.4158E 00	0.0	0.1729E 00	-0.1036E 01	0.0	0.1073E 01
60.0	-0.2558E 00	0.0	0.6542E-01	-0.1391E 01	0.0	0.1935E 01
70.0	-0.1416E 00	0.0	0.2005E-01	-0.1607E 01	0.0	0.2582E 01
80.0	-0.6182E-01	0.0	0.3822E-02	-0.1716E 01	0.0	0.2944E 01
90.0	-0.1058E-06	0.0	0.1119E-13	-0.1748E 01	0.0	0.3057E 01
100.0	0.6182E-01	0.0	0.3822E-02	-0.1716E 01	0.0	0.2944E 01
110.0	0.1416E 00	0.0	0.2005E-01	-0.1607E 01	0.0	0.2582E 01
120.0	0.2558E 00	0.0	0.6542E-01	-0.1391E 01	0.0	0.1935E 01
130.0	0.4158E 00	0.0	0.1729E 00	-0.1036E 01	0.0	0.1073E 01
140.0	0.6218E 00	0.0	0.3867E 00	-0.5331E 00	0.0	0.2842E [illegible]
150.0	0.8558E 00	0.0	0.7324E 00	0.7477E-01	0.0	0.5591E-02
160.0	0.1080E 01	0.0	0.1165E 01	0.6803E 00	0.0	0.4628E [illegible]
170.0	0.1243E 01	0.0	0.1546E 01	0.1134E 01	0.0	0.1287E 01
180.0	0.1304E 01	0.0	0.1699E 01	0.1304E 01	0.0	0.1699E 01

ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA =-0.8736E- 2

θ	REAL(Eθ)	IMAG(Eθ)	GAINθ	REAL(EØ)	IMAG(EØ)	GAINØ
0.0	-0.2266E 01	0.0	0.5136E 01	0.2266E 01	0.0	0.5136E 01
10.0	-0.2176E 01	0.0	0.4736E 01	0.2180E 01	0.0	0.4752E 01
20.0	-0.1929E 01	0.0	0.3722E 01	0.1943E 01	0.0	0.3776E 01
30.0	-0.1584E 01	0.0	0.2510E 01	0.1614E 01	0.0	0.2637E 01
40.0	-0.1210E 01	0.0	0.1465E 01	0.1263E 01	0.0	0.1595E 01
50.0	-0.8632E 00	0.0	0.7452E 00	0.9445E 00	0.0	0.8921E [illegible]
60.0	-0.5722E 00	0.0	0.3275E 00	0.6923E 00	0.0	0.4793E [illegible]
70.0	-0.3416E 00	0.0	0.1167E 00	0.5167E 00	0.0	0.2670E [illegible]
80.0	-0.1582E 00	0.0	0.2501E-01	0.4154E 00	0.0	0.1725E 00
90.0	-0.2768E-06	0.0	0.7661E-13	0.3825E 00	0.0	0.1463E [illegible]
100.0	0.1582E 00	0.0	0.2501E-01	0.4154E 00	0.0	0.1725E [illegible]
110.0	0.3416E 00	0.0	0.1167E 00	0.5167E 00	0.0	0.2670E [illegible]
120.0	0.5722E 00	0.0	0.3275E 00	0.6923E 00	0.0	0.4793E 00
130.0	0.8632E 00	0.0	0.7452E 00	0.9445E 00	0.0	0.8921E 00
140.0	0.1210E 01	0.0	0.1465E 01	0.1263E 01	0.0	0.1595E 01
150.0	0.1584E 01	0.0	0.2510E 01	0.1614E 01	0.0	0.2607E 01
160.0	0.1929E 01	0.0	0.3722E 01	0.1943E 01	0.0	0.3776E 01
170.0	0.2176E 01	0.0	0.4736E 01	0.2180E 01	0.0	0.4752E 01
180.0	0.2266E 01	0.0	0.5136E 01	0.2266E 01	0.0	0.5136E 01

ELECTRIC FIELD AND GAIN OF EIGENCURRENT FOR LAMBDA -0.2645E 02

θ	REAL(Eθ)	IMAG(Eθ)	GAINθ	REAL(EØ)	IMAG(EØ)	GAINØ
0.0	0.1836E 01	0.0	0.3371E 01	-0.1836E 01	0.0	0.3371E 01
10.0	0.1503E 01	0.0	0.2260E 01	-0.1723E 01	0.0	0.2969E 01
20.0	0.6433E 00	0.0	0.4139E 00	-0.1419E 01	0.0	0.2014E 01
30.0	-0.4076E 00	0.0	0.1661E 00	-0.1011E 01	0.0	0.1021E 01
40.0	-0.1289E 01	0.0	0.1661E 01	-0.5953E 00	0.0	0.3544E 00
50.0	-0.1766E 01	0.0	0.3120E 01	-0.2446E 00	0.0	0.5984E-01
60.0	-0.1780E 01	0.0	0.3169E 01	0.1034E-01	0.0	0.1069E-03
70.0	-0.1403E 01	0.0	0.1968E 01	0.1716E 00	0.0	0.2945E-01
80.0	-0.7642E 00	0.0	0.5840E 00	0.2568E 00	0.0	0.6593E-01
90.0	-0.1412E-05	0.0	0.1992E-11	0.2829E 00	0.0	0.8005E-01
100.0	0.7642E 00	0.0	0.5840E 00	0.2568E 00	0.0	0.6593E-01
110.0	0.1403E 01	0.0	0.1968E 01	0.1716E 00	0.0	0.2945E-01
120.0	0.1780E 01	0.0	0.3169E 01	0.1034E-01	0.0	0.1069E-03
130.0	0.1766E 01	0.0	0.3120E 01	-0.2446E 00	0.0	0.5984E-01
140.0	0.1289E 01	0.0	0.1661E 01	-0.5953E 00	0.0	0.3544E 00
150.0	0.4076E 00	0.0	0.1661E 00	-0.1011E 01	0.0	0.1021E 01
160.0	-0.6433E 00	0.0	0.4139E 00	-0.1419E 01	0.0	0.2014E 01
170.0	-0.1503E 01	0.0	0.2260E 01	-0.1723E 01	0.0	0.2969E 01
180.0	-0.1836E 01	0.0	0.3371E 01	-0.1836E 01	0.0	0.3371E 01

VI. SCATTERING CROSS SECTIONS

Program #5 which calculates and plots σ/λ^2 for an axially incident plane wave accepts punched card data according to

```
      READ (1,10) NP, NT, NS, JM, BK
10    FORMAT (4I3, E14.7)
      READ (1,11)(AMD(I), I = 1, JM)
11    FORMAT (5E14.7)
      READ (1,15) (RH(I), I = 1, NP)
      READ (1,15)(ZH(I), I = 1, NP)
15    FORMAT (10F8.4)
      READ (1,50)(L(I), I = 1, JM)
50    FORMAT (20I3)
```

The variables NP, NT, NS, JM, BK, RH and ZH are the same as those in program #4. The variable AMD appears under the heading "eigenvalues of the matrix B" in the output of program #2. The L(I)th eigencurrent is the Ith eigencurrent to be considered in the modal expansion of the scattered field. The variable L(I) is necessary because it is desirable to perform the modal expansion by adding eigencurrents in order of increasing $|\lambda|$. (Program #2 has ordered the eigencurrents in order of increasing λ.) For instance, if λ_6 corresponds to the smallest $|\lambda|$ and λ_5 corresponds to the next smallest $|\lambda|$, then L(1) = 6 and L(2) = 5. The impedance matrix and eigencurrents are read from records 1 and 2 of direct access data set 6 according to

```
      REWIND 6
      READ(6)(Z(I), I = 1, NZ)
      READ(6)(FI(I), I = 1, NZ1)
```

where

```
      NZ = NM2*NM2
      NZ1 = NM2*JM
```

and NM2 is either NP-1 or NP-3 depending on whether or not the generating curve C closes upon itself.

The subroutine PLANE is the same as in program #4. Much of the logic before statement 7 in the main program is devoted to preparing the input data for plane and is thus the same as in program #4.

DO loop 80 modifies the impedance matrix according to (2-47) of [1]. DO loop 19 obtains the matrix X appearing in (2-15) of [1]. Statement 6 inverts the impedance matrix to obtain the admittance matrix Y.

DO loop 85 calculates the scattered field by inserting

$$\begin{bmatrix} I^t \\ \\ I^\phi \end{bmatrix} = [I] = [Y] \left[\widetilde{R_1^{t\theta} \qquad -jR_1^{\phi\theta}} \right] \tag{15}$$

into (5) for n=1. Equation (15) is possible because plane wave excitation and measurement coefficients have the same form. Notice that (1-48) of [1] is written using the eigencurrents as a basis while (15) is written with the expansion functions (2-42) in mind. The excitation of a $\vec{u}_\theta$ polarized plane wave incident in the plane $\phi = 0$ gives rise to expansion functions (2-42) of [1] while the excitation of a $\vec{u}_\phi$ polarized plane wave in the plane $\phi = 0$ gives rise to expansion functions (2-43) of [1]. In DO loop 85, K5=1 obtains plane wave excitation from the direction ($\theta=\pi,\phi=0$) and K5=2 that from the direction ($\theta=0,\phi=0$). DO loop 82 stores the column matrix [I] of (15) in E3. DO loop 103 stores E_θ, E_ϕ, $(\sigma/\lambda^2)_\theta$, and $(\sigma/\lambda^2)_\phi$ in E(J), E(J+NT), SIG(J), and SIG(J+NT) where J indicates the J^{th} measurement polar angle θ. Here, E_θ is the θ component of the far zone scattered field in the plane $\phi = 0$ and E_ϕ is the ϕ component of the scattered field in the plane $\phi = \frac{\pi}{2}$. The phase of the scattered field is normalized to $-je^{-jkr}$. The magnitudes $|E_\theta|$ and $|E_\phi|$ are such that

$$\begin{aligned} (\sigma/\lambda^2)_\theta &= |E_\theta|^2 \\ (\sigma/\lambda^2)_\phi &= |E_\phi|^2 \end{aligned} \tag{16}$$

The constant C1 used to normalize the scattered field is given by

$$C1 = \left(\frac{\omega^2\mu^2}{4\pi\lambda^2}\right)^{1/2} \tag{17}$$

The factor $\omega^2\mu^2/(4\pi\lambda^2)$ appears in (1-51) of [1]. The logic between statements 119 and 109 finds SCL(K5) so that the maximum value of SIG(J)*SCL(K5) for J = 1,2,...2*NT is always between 1.2 and 3.0. DO loop 106 prepares horizontal and vertical coordinates E5 and E6 suitable for plotting σ/λ^2.

DO loop 48 multiplies the eigencurrents by ρ to retrieve the coefficients [I] of the expansion functions (2-42) of [1]. The matrix [I] is stored in FI with subscripts (J-1)*NM2+1 to J*NM2 denoting the J^{th} eigencurrent. The admittance matrix [Y] appearing in (15) will be expressed in terms of [I] whose columns are the expansion coefficients of the eigencurrents. For the moment, assume that there are NM2 eigencurrents so that [I] is a square matrix. Equation (2-18) of [1] can be multiplied by $[\tilde{I}]$ to yield

$$[\tilde{I}][Z][I] = [\tilde{I}][X][I]\ (j + 1/\lambda) \tag{18}$$

Because of the orthogonality relationships (2-24), of [1], the right side of (18) is a diagonal matrix. Equation (18) can be inverted to obtain

$$[\tilde{I}]^{-1}[Y][I]^{-1} = \frac{1}{[\tilde{I}][X][I]\ (j + 1/\lambda)} \tag{19}$$

which leads to

$$[Y] = [I]\ \frac{1}{[\tilde{I}][X][I]\ (j + 1/\lambda)}\ [\tilde{I}] \tag{20}$$

Equation (15) is introduced into (5) with the result

$$\begin{bmatrix} E_\theta \\ E_\phi \end{bmatrix} = \frac{-j\omega\mu e^{-jkr}}{4\pi r} \begin{bmatrix} \cos\phi & 0 \\ 0 & \sin\phi \end{bmatrix} \begin{bmatrix} \hat{R}_1^{t\theta} & -j\hat{R}_1^{\phi\theta} \\ j\hat{R}_1^{t\phi} & \hat{R}_1^{\phi\phi} \end{bmatrix} [Y] \widetilde{\left[\hat{R}_1^{t\theta} \quad -j\hat{R}_1^{\phi\theta} \right]} \tag{21}$$

where [Y] is given by (20). Equation (21) is a specialization of (1-37) of [1]. Alternatively, (21) could have been obtained by finding the matrix $\begin{bmatrix} [I^t] \\ [I^t] \end{bmatrix}$ associated with $\vec{J}$ of (1-30) of [1] and inserting this matrix into (5). Evidently, the use of JM<NM2 terms of the sum (1-37) of [1] is equivalent to using only JM columns of [I] in (20). DO loop 21 stores the diagonal matrix $\dfrac{1}{[\tilde{I}][X][I]\ (j + 1/\lambda)}$ appearing in (20) in E3. The inner DO loop 27 stores

$$[I]\ \frac{1}{[\tilde{I}][X][I]\ (j + 1/\lambda)}$$ in T3.

The index K of DO loop 29 indicates that K columns of [I] are being used. DO loop 30 adds the contribution of the L(K)th column of [I] to the admittance matrix [Y]. In DO loop 65, K4 = 1 obtains the plane wave incident from (θ=π, ϕ=0) and K4=2 that from (θ=0, ϕ=0). Inner DO loop 45 stores $[Y]\widehat{[\hat{R}_1^{t\theta}\ \ -j\hat{R}_1^{\theta\phi}]}$ in E3. In DO loop 66, K5=1 obtains E_θ and K5=2 obtains E_ϕ. DO loop 44 calculates E_θ, E_ϕ, $(\sigma/\lambda^2)_\theta$, and $(\sigma/\lambda^2)_\phi$ and stores them in E and SIG. The quantity σ/λ^2computed using the modal approximation to [Y] is plotted as symbols (X for θ polarization and □ for ϕ polarization) and the quantity σ/λ^2 computed using [Y] obtained by inverting the impedance matrix is plotted as a solid curve in the latter part of DO loop 66. The logic following statement 66 is devoted to drawing the axes for the plot and printing the quantities E_θ, E_φ, $(\sigma/\lambda^2)_\theta$ and $(\sigma/\lambda^2)_\phi$. The scattered field components E_θ, E_ϕ that are printed lack the phase factor $-je^{-jkr}$ and are normalized according to (16).

Minimum allocations are given by

```
COMPLEX Z(NM2*NM2), Y(NM2*NM2), VR(2*NT*NM2), E3(NM2), E(NT*2), T3(NM2*JM)
DIMENSION AMD(JM), RH(NP), ZH(NP), FI(NM2*JM),
          DH(NP-1), TH(NT), X(NM2*NM2), SIG(NT*2),
          L(JM), SN(NT), CS(NT), E9(NT), E10(NT),
          E5(NT*8), E6(NT*8)
```

```
COMMON RS(NP-1), ZS(NP-1), SV(NP-1), T(NM2*2), TR(NM2*2)
DIMENSION BJ(3* (NP-1))
DIMENSION LR(NM2)
```

Here, NP is its value after execution of statement 90 in the main program. Note that BJ appears in PLANE and LR in LINEQ.

Listing of Program #5

```
//                 (0034,EE,4,2,,7),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE PLANE(VVR,THR,NT)
      COMPLEX VVR(1),A5,A6,U
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      DIMENSION BJ(126),THR(1),FK(20)
      KG=NP-1
      NM=KG/2-1
      M2=NN+2
      A5=2.*3.141593*U**(NN+1)
      NV=NM*4
      FK(1)=1.
      DO 153 J=1,M2
      J1=J+1
      FK(J1)=FK(J)*J
  153 CONTINUE
      DO 156 L=1,NT
      L1=(L-1)*NV
      CS=COS(THR(L))
      SN=SIN(THR(L))
      BCS=BK*CS
      DO 302 J=1,KG
      X=RS(J)*BK*SN
      J1=J
      I1=NN
      IF(I1) 303,304,303
  304 I1=I1+1
      J1=J1+KG
  303 DO 305 JJ=I1,M2
      IF(X-1.E-5) 1,1,2
    1 IF(JJ-1) 3,3,4
    3 BJ(J1)=1.
      GO TO 306
    4 BJ(J1)=0.
      GO TO 306
    2 RH=X/2.
      RH2=RH*RH
      RH3=RH**(JJ-1)
      BJ(J1)=RH3/FK(JJ)
      SS=BJ(J1)
    8 SST=SS*1.E-7
      DO 155 K=1,20
      SS=-SS*RH2/K/(K+JJ-1)
      BJ(J1)=BJ(J1)+SS
      IF(ABS(SS)-SST) 306,306,155
  155 CONTINUE
      STOP 155
  306 J1=J1+KG
  305 CONTINUE
  302 CONTINUE
      IF(NN) 307.308.307
  308 DO 309 J=1,KG
      J1=J+2*KG
      BJ(J)=-BJ(J1)
  309 CONTINUE
  307 DO 300 J=1,NM
      J1=J+L1
      J2=J1+NM
      J3=J2+NM
```

```
      J4=J3+NM
      VVR(J1)=0.
      VVR(J2)=0.
      VVR(J3)=0.
      VVR(J4)=0.
      DO 301 I=1,4
      I1=2*(J-1)+I
      I4=4*(J-1)+I
      I2=I1+KG
      I3=I2+KG
      A6=(COS(ZS(I1)*BCS)+U*SIN(ZS(I1)*BCS))*A5
      BJ1=(BJ(I3)+BJ(I1))*.5
      BJ2=(BJ(I3)-BJ(I1))*.5
      VVR(J1)=VVR(J1)+A6*(CS*SV(I1)*BJ2+SN*CV(I1)*BJ(I2)*U)*T(I4)
      VVR(J2)=VVR(J2)+A6*CS*BJ1*TR(I4)
      VVR(J3)=VVR(J3)+A6*SV(I1)*BJ1*T(I4)
      VVR(J4)=VVR(J4)+A6*BJ2*TR(I4)
  301 CONTINUE
  300 CONTINUE
  156 CONTINUE
      RETURN
      END
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(1),STOR,STO,ST,S
      DIMENSION LR(58)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M1+I
      K2=M1+K
      IF(CABS(C(K1))-CABS(C(K2))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
```

```
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
   10 CONTINUE
   11 CONTINUE
      M1=M1+LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
      K2=J2+I
      K1=J1+I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1+LL
    9 CONTINUE
      RETURN
      END
      COMPLEX U,U1,Z(1444),Y(1444),VR(5548),E3(38),E(146),T3(1444)
      DIMENSION AMD(38),RH(41),ZH(41),FI(1444),DH(40),TH(73),X(1444)
      DIMENSION SIG(146),L(38),AREA(400),SN(73),CS(73),E9(73),E10(73)
      DIMENSION XP(2),YP(2),E5(584),E6(584),SCL(2)
      EQUIVALENCE (Z(1),Y(1))
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,NN,T(80),TR(80)
      CALL PLOTS(AREA,400)
      READ(1,10) NP,NT,NS,JM,BK
   10 FORMAT(4I3,E14.7)
      READ(1,11)(AMD(I),I=1,JM)
   11 FORMAT(5E14.7)
      READ(1,15)(RH(I),I=1,NP)
      READ(1,15)(ZH(I),I=1,NP)
   15 FORMAT(10F8.4)
      READ(1,50)(L(I),I=1,JM)
   50 FORMAT(20I3)
      WRITE(3,9)
    9 FORMAT('1 NP NT NS JM       BK')
      WRITE(3,12) NP,NT,NS,JM,BK
   12 FORMAT(1X,4I3,E14.7)
      WRITE(3,13)(AMD(I),I=1,JM)
   13 FORMAT('0AMD'/(1X,5E14.7))
      WRITE(3,16)(RH(I),I=1,NP)
   16 FORMAT('0RH'/(1X,10F8.4))
      WRITE(3,127)(ZH(I),I=1,NP)
  127 FORMAT('0ZH'/(1X,10F8.4))
      WRITE(3,51)(L(I),I=1,JM)
   51 FORMAT('0L'/(1X,20I3))
      PI=3.141593
      ETA=376.730
      U=(0.,1.)
      C1=BK*BK*ETA/(4.*SQRT(PI**3))
      KL=1
```

```
      IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 90
      KL=0
      RH(NP+1)=RH(2)
      ZH(NP+1)=ZH(2)
      RH(NP+2)=RH(3)
      ZH(NP+2)=ZH(3)
      NP=NP+2
   90 NM2=NP-3
      NZ=NM2*NM2
      NZ1=NM2*JM
      REWIND 6
      READ(6)(Z(I),I=1,NZ)
      READ(6)(FI(I),I=1,NZ1)
      NM=NM2/2
      NM4=NM*4
      NT2=NT*2
      NT3=NT-NS
      NT4=NS+1
      DO 40 I=2,NP
      I2=I-1
      RR1=RH(I)-RH(I2)
      RR2=ZH(I)-ZH(I2)
      DH(I2)=SQRT(RR1*RR1+RR2*RR2)
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      SV(I2)=RR1/DH(I2)
      CV(I2)=RR2/DH(I2)
   40 CONTINUE
      DO 41 J=1,NM
      J2=2*(J-1)+1
      J3=J2+1
      J4=J3+1
      J5=J4+1
      J6=4*(J-1)+1
      J7=J6+1
      J8=J7+1
      J9=J8+1
      DEL1=DH(J2)+DH(J3)
      DEL2=DH(J4)+DH(J5)
      T(J6)=DH(J2)*DH(J2)/2./DEL1
      T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
      T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
      T(J9)=DH(J5)*DH(J5)/2./DEL2
   41 CONTINUE
      DO 91 J=1,NM4
      TR(J)=T(J)
   91 CONTINUE
      IF(KL.EQ.0) GO TO 95
      IF(RH(1))93,94,93
   93 DEL1=DH(1)+DH(2)
      TR(1)=DH(1)*(1.+(DH(2)+DH(1)/2.)/DEL1)
      TR(2)=DH(2)*(1.+DH(2)/2./DEL1)
   94 IF(RH(NP))96,95,96
   96 J1=(NM-1)*4+3
      J2=J1+1
      DEL2=DH(NP-2)+DH(NP-1)
      TR(J1)=DH(NP-2)*(1.+DH(NP-2)/2./DEL2)
      TR(J2)=DH(NP-1)*(1.+(DH(NP-2)+DH(NP-1)/2.)/DEL2)
   95 DEL=PI/(NT-1)
      DO 43 J=1,NT
```

```
      TH(J)=(J-1)*DEL
      SN(J)=SIN(TH(J))
      CS(J)=COS(TH(J))
   43 CONTINUE
      NN=1
    7 CALL PLANE(VR,TH,NT)
      DEL1=180./PI
      DO 128 J=1,NT
      TH(J)=TH(J)*DEL1
  128 CONTINUE
      XP(1)=2.
      XP(2)=8.
      YP(1)=5.
      YP(2)=5.
      U1=.5*U
      DO 80 J=1,NM
      J1=(J-1)*NM2+NM
      J4=(J+NM-1)*NM2
      DO 81 I=1,NM
      J2=J1+I
      J3=J4+I
      Z(J2)=U1*Z(J2)
      Z(J3)=-U1*Z(J3)
      J5=J2-NM
      J6=J3+NM
      Z(J5)=.5*Z(J5)
      Z(J6)=.5*Z(J6)
   81 CONTINUE
   80 CONTINUE
      DO 19 J=1,NM2
      J2=(J-1)*NM2
      DO 20 I=1,J
      J3=J2+I
      J4=(I-1)*NM2+J
      U1=.5*(Z(J3)+Z(J4))
      X(J3)=AIMAG(U1)
      X(J4)=X(J3)
   20 CONTINUE
   19 CONTINUE
    6 CALL LINEQ(NM2,Z)
      DO 85 K5=1,2
      J5=(2-K5)*(NT-1)*NM4
      DO 82 I=1,NM2
      E3(I)=0.
      DO 83 KK=1,NM2
      J3=I+(KK-1)*NM2
      J4=J5+KK
      E3(I)=E3(I)+Z(J3)*VR(J4)
   83 CONTINUE
   82 CONTINUE
      S2=0.
      DO 103 K6=1,2
      J5=(K6-1)*NM2
      J8=(K6-1)*NT
      DO 84 J=1,NT
      J7=J+J8
      U1=0.
      J1=(J-1)*NM4+J5
      DO 88 I=1,NM2
      J2=J1+I
```

```
      U1=U1+VR(J2)*E3(I)
   88 CONTINUE
      E(J7)=C1*U1
      S1=CABS(E(J7))
      SIG(J7)=S1*S1
      IF(SIG(J7).GT.S2) S2=SIG(J7)
   84 CONTINUE
  103 CONTINUE
      WRITE(3,5)
    5 FORMAT('OSCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH S
     1QUARED')
      S1=(2-K5)*180.
      WRITE(3,124) S1
  124 FORMAT(' BY MATRIX INVERSION, INCIDENCE FROM O=',F4.0,', O=0')
    2 WRITE(3,118)
  118 FORMAT('+',36X,'-',7X,'/')
      WRITE(3,39)
   39 FORMAT('0    O      REAL(EO)     IMAG(EO)   SO/(LAM)**2    REAL(EO)
     1IMAG(EO)   SO/(LAM)**2')
      WRITE(3,117)
  117 FORMAT('+    -',11X,'-',11X,'-     -',18X,'/',11X,'/     /')
      DO 37 J=1,NT,NS
      J1=J+NT
      WRITE(3,38) TH(J),E(J),SIG(J),E(J1),SIG(J1)
   38 FORMAT(1X,F6.1,6E12.4)
   37 CONTINUE
  119 J1=10+ALOG10(S2)
      S3=.1**(J1-10)
      S4=S2*S3
      IF(S4-1.5) 110,110,111
  110 SCL(K5)=2.*S3
      GO TO 112
  111 IF(S4-3.) 113,113,114
  113 SCL(K5)=S3
      GO TO 112
  114 IF(S4-6.) 115,115,116
  115 SCL(K5)=.5*S3
      GO TO 112
  116 SCL(K5)=.2*S3
  112 S5=1./SCL(K5)
      WRITE(3,109) S5
  109 FORMAT('OONE INCH CORRESPONDS TO SIGMA/(LAMBDA)**2=',E11.4)
      DO 106 K6=1,2
      J1=(K6-1)*NT
      J3=NT2*((K5-1)*2+K6-1)
      DO 107 J=1,NT
      J7=J1+J
      J8=J+J3
      J9=J3+NT2-J+1
      S2=SIG(J7)*SCL(K5)
      S1=SN(J)*S2
      E5(J8)=5.+S1
      E5(J9)=5.-S1
      E6(J8)=5.+CS(J)*S2
      E6(J9)=E6(J8)
  107 CONTINUE
  106 CONTINUE
   85 CONTINUE
      DO 48 J=1,JM
      J1=(J-1)*NM2
```

```
      DO 18 I=1,NM
      J2=J1+I
      J3=J2+NM
      J4=2*I+1
      FI(J2)=FI(J2)*RH(J4)
      FI(J3)=FI(J3)*RH(J4)
   18 CONTINUE
   48 CONTINUE
      DO 21 J=1,JM
      J1=(J-1)*NM2
      S1=0.
      DO 22 I=1,NM2
      S2=0.
      J4=(I-1)*NM2
      DO 23 K=1,NM2
      J3=J1+K
      J2=J4+K
      S2=S2+X(J2)*FI(J3)
   23 CONTINUE
      J2=J1+I
      S1=S1+S2*FI(J2)
   22 CONTINUE
      E3(J)=1./S1/(U+1./AMD(J))
      DO 27 I=1,NM2
      J2=J1+I
      T3(J2)=FI(J2)*E3(J)
   27 CONTINUE
   21 CONTINUE
      DO 32 J=1,N2
      Y(J)=0.
   32 CONTINUE
      DO 29 K=1,JM
   52 J3=(L(K)-1)*NM2
      DO 30 J=1,NM2
      J1=(J-1)*NM2
      J5=J3+J
      DO 31 I=1,J
      J2=J1+I
      J4=J3+I
      Y(J2)=Y(J2)+FI(J5)*T3(J4)
      J6=(I-1)*NM2+J
      Y(J6)=Y(J2)
   31 CONTINUE
   30 CONTINUE
      DO 65 K4=1,2
      J8=(2-K4)*(NT-1)*NM4
      DO 45 I=1,NM2
      E3(I)=0.
      DO 46 KK=1,NM2
      J3=I+(KK-1)*NM2
      J4=J8+KK
      E3(I)=E3(I)+Y(J3)*VR(J4)
   46 CONTINUE
   45 CONTINUE
      DO 66 K5=1,2
      K3=((K4-1)*2+K5-1)*NT2
      J7=(K5-1)*NT
      K2=(2-K5)*4
      J5=(K5-1)*NM2
      DO 44 J=1,NT,NS
```

```
      U1=0.
      J1=(J-1)*NM4+J5
      DO 47 I=1,NM2
      J2=J1+I
      U1=U1+VR(J2)*E3(I)
   47 CONTINUE
      J8=J7+J
      E(J8)=C1*U1
      S2=CABS(E(J8))
      SIG(J8)=S2*S2
      S2=SIG(J8)*SCL(K4)
      E9(J)=5.+SN(J)*S2
      E10(J)=5.+CS(J)*S2
   44 CONTINUE
      DO 86 J=1,NT,NS
      CALL SYMBOL(E9(J),E10(J),.07,K2,0.,-1)
   86 CONTINUE
      DO 89 J=NT4,NT3,NS
      J1=NT-J+1
      S1=10.-E9(J1)
      CALL SYMBOL(S1,E10(J1),.07,K2,0.,-1)
   89 CONTINUE
      CALL LINE(E5(K3),E6(K3),NT2,1,0,0)
   66 CONTINUE
   68 CALL LINE(XP,YP,2,1,0,0)
      DO 77 J=1,7
      S1=9-J
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
   77 CONTINUE
      CALL LINE(YP,XP,2,1,0,0)
      DO 78 J=1,7
      S1=9-J
      CALL SYMBOL(5.,S1,.14,13,90.,-1)
   78 CONTINUE
      CALL PLOT(7.,0.,-3)
      WRITE(3,122) K
  122 FORMAT('0',I3,' MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/
     1WAVELENGTH SQUARED')
      S1=(2-K4)*180.
      WRITE(3,129) S1
  129 FORMAT(' INCIDENCE FROM 0=',F4.0,', 0=0')
      WRITE(3,120)
  120 FORMAT('+',15X,'-',7X,'/')
      WRITE(3,39)
      WRITE(3,117)
      DO 126 J=1,NT,NS
      J1=J+NT
      WRITE(3,38) TH(J),E(J),SIG(J),E(J1),SIG(J1)
  126 CONTINUE
   65 CONTINUE
   29 CONTINUE
      CALL PLOT(6.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1.DISP=OLD,UNIT=2314,                X
//             VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
 21 73  4  3 0.3141593E+00
 0.2559665E+01-0.8785684E-02-0.2644527E+02

  0.0000  0.5000  1.0000  1.5000  2.0000  2.5000  3.0000  3.5000  4.0000  4.5000
  5.0000  5.5000  6.0000  6.5000  7.0000  7.5000  8.0000  8.5000  9.0000  9.5000
 10.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000
  2  1  3
/*
```

Output of Program #5

NP	NT	NS	JM	BK
21	73	4	3	0.3141593E 01

AMD

0.2559665E 01-0.8785684E-02-0.2644527E 02

RH

0.0	0.5000	1.0000	1.5000	2.0000	2.5000	3.0000	3.5000	4.0000	4.5000
5.0000	5.5000	6.0000	6.5000	7.0000	7.5000	8.0000	8.5000	9.0000	9.5000
10.0000									

ZH

0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0									

L

2 1 3

SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
BY MATRIX INVERSION, INCIDENCE FROM θ=180., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	-0.3024E 01	.2301E 00	0.9198E 01	0.3024E 01	-0.2301E 00	0.9198E 01
10.0	-0.2903E 01	0.2290E 00	0.8478E 01	0.2897E 01	-0.1925E 00	0.8429E 01
20.0	-0.2570E 01	0.2243E 00	0.6655E 01	0.2550E 01	-0.9186E-01	0.6510E 01
30.0	-0.2106E 01	0.2130E 00	0.4480E 01	0.2071E 01	0.4138E-01	0.4289E 01
40.0	-0.1604E 01	0.1929E 00	0.2611E 01	0.1561E 01	0.1736E 00	0.2468E 01
50.0	-0.1140E 01	0.1636E 00	0.1327E 01	0.1105E 01	0.2812E 00	0.1301E 01
60.0	-0.7531E 00	0.1272E 00	0.5833E 00	0.7484E 00	0.3552E 00	0.6863E 00
70.0	-0.4479E 00	0.8638E-01	0.2081E 00	0.5029E 00	0.3988E 00	0.4120E 00
80.0	-0.2069E 00	0.4351E-01	0.4469E-01	0.3627E 00	0.4200E 00	0.3079E 00
90.0	-0.3617E-06	0.7842E-07	0.1370E-12	0.3175E 00	0.4261E 00	0.2824E 00
100.0	0.2069E 00	-0.4351E-01	0.4469E-01	0.3627E 00	0.4200E 00	0.3079E 0
110.0	0.4479E 00	-0.8638E-01	0.2081E 00	0.5029E 00	0.3988E 00	0.4120E 00
120.0	0.7531E 00	-0.1272E 00	0.5833E 00	0.7484E 00	0.3552E 00	0.6863E 00
130.0	0.1140E 01	-0.1636E 00	0.1327E 01	0.1105E 01	0.2812E 00	0.1301E 01
140.0	0.1604E 01	-0.1929E 00	0.2611E 01	0.1561E 01	0.1736E 00	0.2468E 01
150.0	0.2106E 01	-0.2130E 00	0.4480E 01	0.2071E 01	0.4138E-01	0.4289E 01
160.0	0.2570E 01	-0.2243E 00	0.6655E 01	0.2550E 01	-0.9186E-01	0.6510E 01
170.0	0.2903E 01	-0.2290E 00	0.8478E 01	0.2897E 01	-0.1925E 00	0.8429E 01
180.0	0.3024E 01	-0.2301E 00	0.9198E 01	0.3024E 01	-0.2301E 00	0.9198E 01

ONE INCH CORRESPONDS TO SIGMA/(LAMBDA)**2= 0.5000E 01

SCATT[illegible]D FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
BY M[illegible]IX INVERSION, INCIDENCE FROM θ= 0., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	0.3024E 01	-0.2301E 00	0.9198E 01	-0.3024E 01	0.2301E 00	0.9198E 01
10.0	0.2903E 01	-0.2290E 00	0.8478E 01	-0.2897E 01	0.1925E 00	0.8429E 01
20.0	0.2570E 01	-0.2243E 00	0.6655E 01	-0.2550E 01	0.9186E-01	0.6510E 01
30.0	0.2106E 01	-0.2130E 00	0.4480E 01	-0.2071E 01	-0.4138E-01	0.4289E 01
40.0	0.1604E 01	-0.1929E 00	0.2611E 01	-0.1561E 01	-0.1736E 00	0.2468E 01
50.0	0.1140E 01	-0.1636E 00	0.1327E 01	-0.1105E 01	-0.2812E 00	0.1301E 01
60.0	0.7531E 00	-0.1272E 00	0.5833E 00	-0.7484E 00	-0.3552E 00	0.6863E 00
70.0	0.4479E 00	-0.8638E-01	0.2081E 00	-0.5029E 00	-0.3988E 00	0.4120E 00
80.0	0.2069E 00	-0.4351E-01	0.4469E-01	-0.3627E 00	-0.4200E 00	0.3079E 00
90.0	0.3617E-06	-0.7842E-07	0.1370E-12	-0.3175E 00	-0.4261E 00	0.2824E 00
100.0	-0.2069E 00	0.4351E-01	0.4469E-01	-0.3627E 00	-0.4200E 00	0.3079E 00
110.0	-0.4479E 00	0.8638E-01	0.2081E 00	-0.5029E 00	-0.3988E 00	0.4120E 00

120.0	-0.7531E 00	0.1272E 00	0.5833E 00	-0.7484E 00	-0.3552E 00	0.6863E 00
130.0	-0.1140E 01	0.1636E 00	0.1327E 01	-0.1105E 01	-0.2812E 00	0.1301E 01
140.0	-0.1604E 01	0.1929E 00	0.2611E 01	-0.1561E 01	-0.1736E 00	0.2468E 01
150.0	-0.2106E 01	0.2130E 00	0.4480E 01	-0.2071E 01	-0.4138E-01	0.4289E 01
160.0	-0.2570E 01	0.2243E 00	0.6655E 01	-0.2550E 01	0.9186E-01	0.6510E 01
170.0	-0.2903E 01	0.2290E 00	0.8478E 01	-0.2897E 01	0.1925E 00	0.8429E 01
180.0	-0.3024E 01	0.2301E 00	0.9198E 01	-0.3024E 01	0.2301E 00	0.9198E 01

ONE INCH CORRESPONDS TO SIGMA/(LAMBDA)**2= 0.5000E 01

1 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM θ=180., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	-0.2895E 01	-0.2543E-01	0.8381E 01	0.2895E 01	0.2543E-01	0.8381E 01
10.0	-0.2780E 01	-0.2442E-01	0.7728E 01	0.2784E 01	0.2446E-01	0.7753E 01
20.0	-0.2464E 01	-0.2165E-01	0.6073E 01	0.2482E 01	0.2181E-01	0.6161E 01
30.0	-0.2023E 01	-0.1778E-01	0.4095E 01	0.2062E 01	0.1812E-01	0.4253E 01
40.0	-0.1546E 01	-0.1358E-01	0.2390E 01	0.1613E 01	0.1417E-01	0.2602E 01
50.0	-0.1103E 01	-0.9687E-02	0.1216E 01	0.1206E 01	0.1060E-01	0.1456E 01
60.0	-0.7309E 00	-0.6422E-02	0.5343E 00	0.8843E 00	0.7769E-02	0.7820E 00
70.0	-0.4363E 00	-0.3833E-02	0.1904E 00	0.6600E 00	0.5799E-02	0.4357E 00
80.0	-0.2020E 00	-0.1775E-02	0.4081E-01	0.5305E 00	0.4661E-02	0.2815E 00
90.0	-0.3535E-06	-0.3106E-08	0.1250E-12	0.4885E 00	0.4292E-02	0.2387E 00
100.0	0.2020E 00	0.1775E-02	0.4081E-01	0.5305E 00	0.4661E-02	0.2815E 00
110.0	0.4363E 00	0.3833E-02	0.1904E 00	0.6600E 00	0.5799E-02	0.4357E 00
120.0	0.7309E 00	0.6422E-02	0.5343E 00	0.8843E 00	0.7769E-02	0.7820E 00
130.0	0.1103E 01	0.9687E-02	0.1216E 01	0.1206E 01	0.1060E-01	0.1456E 01
140.0	0.1546E 01	0.1358E-01	0.2390E 01	0.1613E 01	0.1417E-01	0.2602E 01
150.0	0.2023E 01	0.1778E-01	0.4095E 01	0.2062E 01	0.1812E-01	0.4253E 01
160.0	0.2464E 01	0.2165E-01	0.6073E 01	0.2482E 01	0.2181E-01	0.6161E 01
170.0	0.2780E 01	0.2442E-01	0.7728E 01	0.2784E 01	0.2446E-01	0.7753E 01
180.0	0.2895E 01	0.2543E-01	0.8381E 01	0.2895E 01	0.2543E-01	0.8381E 01

1 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM θ= 0., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	0.2895E 01	0.2543E-01	0.8381E 01	-0.2895E 01	-0.2543E-01	0.8381E 01
10.0	0.2780E 01	0.2442E-01	0.7728E 01	-0.2784E 01	-0.2446E-01	0.7753E 01
20.0	0.2464E 01	0.2165E-01	0.6073E 01	-0.2482E 01	-0.2181E-01	0.6161E 01
30.0	0.2023E 01	0.1778E-01	0.4095E 01	-0.2062E 01	-0.1812E-01	0.4253E 01
40.0	0.1546E 01	0.1358E-01	0.2390E 01	-0.1613E 01	-0.1417E-01	0.2602E 01
50.0	0.1103E 01	0.9687E-02	0.1216E 01	-0.1206E 01	-0.1060E-01	0.1456E 01
60.0	0.7309E 00	0.6422E-02	0.5343E 00	-0.8843E 00	-0.7769E-02	0.7820E 00
70.0	0.4363E 00	0.3833E-02	0.1904E 00	-0.6600E 00	-0.5799E-02	0.4357E 00
80.0	0.2020E 00	0.1775E-02	0.4081E-01	-0.5305E 00	-0.4661E-02	0.2815E 00
90.0	0.3535E-06	0.3106E-08	0.1250E-12	-0.4885E 00	-0.4292E-02	0.2387E 00
100.0	-0.2020E 00	-0.1775E-02	0.4081E-01	-0.5305E 00	-0.4661E-02	0.2815E 00
110.0	-0.4363E 00	-0.3833E-02	0.1904E 00	-0.6600E 00	-0.5799E-02	0.4357E 00
120.0	-0.7309E 00	-0.6422E-02	0.5343E 00	-0.8843E 00	-0.7769E-02	0.7820E 00
130.0	-0.1103E 01	-0.9687E-02	0.1216E 01	-0.1206E 01	-0.1060E-01	0.1456E 01
140.0	-0.1546E 01	-0.1358E-01	0.2390E 01	-0.1613E 01	-0.1417E-01	0.2602E 01
150.0	-0.2023E 01	-0.1778E-01	0.4095E 01	-0.2062E 01	-0.1812E-01	0.4253E 01
160.0	-0.2464E 01	-0.2165E-01	0.6073E 01	-0.2482E 01	-0.2181E-01	0.6161E 01
170.0	-0.2780E 01	-0.2442E-01	0.7728E 01	-0.2784E 01	-0.2446E-01	0.7753E 01
180.0	-0.2895E 01	-0.2543E-01	0.8381E 01	-0.2895E 01	-0.2543E-01	0.8381E 01

2 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM θ=180., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	-0.3022E 01	0.2994E 00	0.9221E 01	0.3022E 01	-0.2994E 00	0.9221E 01
10.0	-0.2901E 01	0.2854E 00	0.8496E 01	0.2895E 01	-0.2582E 00	0.8446E 01
20.0	-0.2569E 01	0.2473E 00	0.6663E 01	0.2548E 01	-0.1477E 00	0.6516E 01
30.0	-0.2107E 01	0.1955E 00	0.4477E 01	0.2069E 01	-0.5127 -03	0.4283E 01
40.0	-0.1607E 01	0.1414E 00	0.2601E 01	0.1561E 01	0.1470E 00	0.2459E 01
50.0	-0.1143E 01	0.9392E-01	0.1315E 01	0.1106E 01	0.2687E 00	0.1295E 01
60.0	-0.7558E 00	0.5731E-01	0.5746E 00	0.7489E 00	0.3543E 00	0.6864E 00
70.0	-0.4501E 00	0.3144E-01	0.2036E 00	0.5036E 00	0.4061E 00	0.4186E 00
80.0	-0.2080E 00	0.1363E-01	0.4347E-01	0.3635E 00	0.4322E 00	0.3189E 00
90.0	-0.3638E-06	0.2325E-07	0.1329E-12	0.3183E 00	0.4399E 00	0.2949E 00
100.0	0.2080E 00	-0.1363E-01	0.4346E-01	0.3635E 00	0.4322E 00	0.3189E 00
110.0	0.4501E 00	-0.3144E-01	0.2036E 00	0.5036E 00	0.4061E 00	0.4186E 00
120.0	0.7559E 00	-0.5731E-01	0.5746E 00	0.7489E 00	0.3543E 00	0.6864E 00
130.0	0.1143E 01	-0.9392E-01	0.1315E 01	0.1106E 01	0.2687E 00	0.1295E 01
140.0	0.1607E 01	-0.1414E 00	0.2601E 01	0.1561E 01	0.1470E 00	0.2459E 01
150.0	0.2107E 01	-0.1955E 00	0.4477E 01	0.2069E 01	-0.5123E-03	0.4283E 01
160.0	0.2569E 01	-0.2473E 00	0.6663E 01	0.2548E 01	-0.1477E 00	0.6515E 01
170.0	0.2901E 01	-0.2854E 00	0.8496E 01	0.2895E 01	-0.2582E 00	0.8446E 01
180.0	0.3022E 01	-0.2994E 00	0.9221E 01	0.3022E 01	-0.2994E 00	0.9221E 01

2 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM θ= 0., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	0.3022E 01	-0.2994E 00	0.9221E 01	-0.3022E 01	0.2994E 00	0.9221E 01
10.0	0.2901E 01	-0.2854E 00	0.8496E 01	-0.2895E 01	0.2582E 00	0.8446E 01
20.0	0.2569E 01	-0.2473E 00	0.6663E 01	-0.2548E 01	0.1477E 00	0.6516E 01
30.0	0.2107E 01	-0.1955E 00	0.4477E 01	-0.2069E 01	0.5127E-03	0.4283E 01
40.0	0.1607E 01	-0.1414E 00	0.2601E 01	-0.1561E 01	-0.1470E 00	0.2459E 01
50.0	0.1143E 01	-0.9392E-01	0.1315E 01	-0.1106E 01	-0.2687E 00	0.1295E 01
60.0	0.7558E 00	-0.5731E-01	0.5746E 00	-0.7489E 00	-0.3543E 00	0.6864E 00
70.0	0.4501E 00	-0.3144E-01	0.2036E 00	-0.5036E 00	-0.4061E 00	0.4186E 00
80.0	0.2080E 00	-0.1363E-01	0.4347E-01	-0.3635E 00	-0.4322E 00	0.3189E 00
90.0	0.3638E-06	-0.2325E-07	0.1329E-12	-0.3183E 00	-0.4399E 00	0.2949E 00
100.0	-0.2080E 00	0.1363E-01	0.4346E-01	-0.3635E 00	-0.4322E 00	0.3189E 00
110.0	-0.4501E 00	0.3144E-01	0.2036E 00	-0.5036E 00	-0.4061E 00	0.4186E 00
120.0	-0.7558E 00	0.5731E-01	0.5746E 00	-0.7489E 00	-0.3543E 00	0.6864E 00
130.0	-0.1143E 01	0.9392E-01	0.1315E 01	-0.1106E 01	-0.2687E 00	0.1295E 01
140.0	-0.1607E 01	0.1414E 00	0.2601E 01	-0.1561E 01	-0.1470E 00	0.2459E 01
150.0	-0.2107E 01	0.1955E 00	0.4477E 01	-0.2069E 01	0.5123E-03	0.4283E 01
160.0	-0.2569E 01	0.2473E 00	0.6663E 01	-0.2548E 01	0.1477E 00	0.6515E 01
170.0	-0.2901E 01	0.2854E 00	0.8496E 01	-0.2895E 01	0.2582E 00	0.8446E 01
180.0	-0.3022E 01	0.2994E 00	0.9221E 01	-0.3022E 01	0.2994E 00	0.9221E 01

3 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM θ=180., Ø=0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(EØ)	IMAG(EØ)	SØ/(LAM)**2
0.0	-0.3024E 01	0.2268E 00	0.9199E 01	0.3024E 01	-0.2268E 00	0.9199E 01
10.0	-0.2903E 01	0.2259E 00	0.8479E 01	0.2897E 01	-0.1931E 00	0.8431E 01
20.0	-0.2570E 01	0.2219E 00	0.6656E 01	0.2550E 01	-0.9159E-01	0.6513E 01
30.0	-0.2105E 01	0.2116E 00	0.4481E 01	0.2071E 01	0.3994E-01	0.4291E 01
40.0	-0.1605E 01	0.1923E 00	0.2612E 01	0.1562E 01	0.1705E 00	0.2469E 01
50.0	-0.1140E 01	0.1637E 00	0.1327E 01	0.1106E 01	0.2784E 00	0.1301E 01
60.0	-0.7532E 00	0.1277E 00	0.5836E 00	0.7489E 00	0.3539E 00	0.6961E 00
70.0	-0.4480E 00	0.8690E-01	0.2082E 00	0.5034E 00	0.3993E 00	0.4129E 00
80.0	-0.2069E 00	0.4384E-01	0.4473E-01	0.3631E 00	0.4220E 00	0.31[illegible]
90.0	-0.3617E-06	0.7905E-07	0.1371E-12	0.3179E 00	0.4287E 00	0.2849E 00
100.0	0.2069E 00	-0.4384E-01	0.4473E-01	0.3631E 00	0.4220E 00	0.31[illegible]
110.0	0.4480E 00	-0.3690E-01	0.2082E 00	0.5034E 00	0.3993E 00	0.4129E 00

120.0	0.7532E 00	-0.1277E 00	0.5836E 00	0.7489E 00	0.3539E 00	0.6861E 00
130.0	0.1140E 01	-0.1637E 00	0.1327E 01	0.1106E 01	0.2784E 00	0.1301E 01
140.0	0.1605E 01	-0.1923E 00	0.2612E 01	0.1562E 01	0.1705E 00	0.2469E 01
150.0	0.2106E 01	-0.2116E 00	0.4481E 01	0.2071E 01	0.3944E-01	0.4291E 01
160.0	0.2570E 01	-0.2219E 00	0.6656E 01	0.2550E 01	-0.9159E-01	0.6513E 01
170.0	0.2903E 01	-0.2259E 00	0.8479E 01	0.2897E 01	-0.1901E 0	0.8431E 01
180.0	0.3024E 01	-0.2268E 00	0.9199E 01	0.3024E 01	-0.2268E 00	0.9199E 01

3 MODE SCATTERED FIELD AND SCATTERING CROSS SECTION/WAVELENGTH SQUARED
INCIDENCE FROM θ= 0., φ= 0

θ	REAL(Eθ)	IMAG(Eθ)	Sθ/(LAM)**2	REAL(Eφ)	IMAG(Eφ)	Sφ/(LAM)**2
0.0	0.3024E 01	-0.2268E 00	0.9199E 01	-0.3024E 01	0.2268E 00	0.9199E 01
10.0	0.2903E 01	-0.2259E 00	0.8479E 01	-0.2897E 01	0.1901E 00	0.8431E 01
20.0	0.2570E 01	-0.2219E 00	0.6656E 01	-0.2550E 01	0.9159E-01	0.6513E 01
30.0	0.2106E 01	-0.2116E 00	0.4481E 01	-0.2071E 01	-0.3944E-01	0.4291E 01
40.0	0.1605E 01	-0.1923E 00	0.2612E 01	-0.1562E 01	-0.1705E 00	0.2469E 01
50.0	0.1140E 01	-0.1637E 00	0.1327E 01	-0.1106E 01	-0.2784E 00	0.1301E 01
60.0	0.7532E 00	-0.1277E 00	0.5836E 00	-0.7489E 00	-0.3539E 00	0.6861E 00
70.0	0.4480E 00	-0.3690E-01	0.2082E 00	-0.5034E 00	-0.3993E 00	0.4129E 00
80.0	0.2069E 00	-0.4384E-01	0.4473E-01	-0.3631E 00	-0.4220E 00	0.3100E 00
90.0	0.3617E-06	-0.7905E-07	0.1371E-12	-0.3179E 00	-0.4287E 00	0.2849E 00
100.0	-0.2069E 00	0.4384E-01	0.4473E-01	-0.3631E 00	-0.4220E 00	0.3100E 00
110.0	-0.4480E 00	0.3690E-01	0.2082E 00	-0.5034E 00	-0.3993E 00	0.4129E 00
120.0	-0.7532E 00	0.1277E 00	0.5836E 00	-0.7489E 00	-0.3539E 00	0.6861E 00
130.0	-0.1140E 01	0.1637E 00	0.1327E 01	-0.1106E 01	-0.2784E 00	0.1301E 01
140.0	-0.1605E 01	0.1923E 00	0.2612E 01	-0.1562E 01	-0.1705E 00	0.2469E 01
150.0	-0.2106E 01	0.2116E 00	0.4481E 01	-0.2071E 01	-0.3944E-01	0.4291E 01
160.0	-0.2570E 01	0.2219E 00	0.6656E 01	-0.2550E 01	0.9159E-01	0.6513E 01
170.0	-0.2903E 01	0.2259E 00	0.8479E 01	-0.2897E 01	0.1901E 00	0.8431E 01
180.0	-0.3024E 01	0.2268E 00	0.9199E 01	-0.3024E 01	0.2268E 00	0.9199E 01

VII. GAIN PATTERNS

Program #6 which calculates and plots the gain pattern for radiation from axially symmetric excitation on a surface of revolution accepts punched card data according to

```
          READ(1,10) NP, NT, NS, JM, NC, NV, BK, SCL
    10    FORMAT(6I3, 2E14.7)
          READ(1,11)(AMD(I), I = 1, JM)
    11    FORMAT (5E14.7)
          READ(1,15)(RH(I), I = 1, NP)
          READ(1,15)(ZH(I), I = 1, NP)
    15    FORMAT (10F8.4)
          READ(1,50)(L(I),I = 1, NC)
    50    FORMAT (20I3)
          READ(1,57)(V(I), I = 1, NV)
    57    FORMAT (7E11.4)
          READ(1,50)(LV(I), I = 1, NV)
```

The variables NP, NT, NS, JM, BK, AMD, RH, ZH, and L are the same as in program #5 except that L now applies only to the $\vec{u}_t$ directed eigencurrents. The output of program #2 characterizes each eigencurrent by NM2 numbers. For the axially symmetric mode, either the first NM2/2 numbers (corresponding to J_t) or the last NM2/2 numbers (corresponding to J_ϕ) are supposed to be zero. Thus L(I) indicates that the L(I)th eigencurrent (only $\vec{u}_t$ directed eigencurrents being considered) to come out of program #2 will be the Ith current to be added to the modal expansion. There are NV $\leq$ NM axially symmetric slots on the body of revolution. The voltage across the Ith slot is V(I). Note that V(I) is complex. The Ith slot is located at the (2*LV(1)+1)th data point (ρ = RH, z = ZH) on the generating curve C. Accordingly, the Ith slot occurs at the peak of the LV(I)th triangle function of $f_{LV(1)}$. The function f_i appears in (2-42) of [1]. For the plots, one inch corresponds to a gain of 1./SCL.

The impedance matrix and eigencurrents are read from the third and fourth records of direct access data set 6 according to

```
REWIND 6
READ (6)
READ (6)
READ (6) (Z(I), I = 1, NZ)
READ (6) (FI(I), I = 1, NZ1)
```

Only half of the impedance matrix is read because only $[Z_o^{tt}]$ of (2-46) of [1] is needed. Program #1 has stored the impedance matrix on the third record of data set 6 columnwise in the block diagonal form given by

$$[Z_o] = \begin{bmatrix} [Z_o^{tt}] & [0] \\ & \\ [0] & [Z_o^{\phi\phi}] \end{bmatrix} \tag{22}$$

The subroutine PLANE is similar to the one compiled with program #5. At present, only $\hat{R}_o^{t\theta}$ of (5) is needed because a slot voltage, corresponding to a $\vec{u}_t$ directed axially symmetric aperture electric field, induces only a $\vec{u}_t$ directed electric current which radiates only a u_θ directed far field.

DO loop 60 suppresses the lower left zero submatrix on the right hand side of (22). The net result of DO loops 71 and 73 is to arrange the eigencurrents (FI) and their eigenvalues (AMD) in the order dictated by L and to suppress intervening zeros from FI. DO loop 62 obtains the matrix [X] appearing in (2-15) of [1] by taking the imaginary part of the average of corresponding off diagonal elements of the impedance matrix. Statement 79 inverts the impedance matrix to obtain the admittance matrix. DO loop 82 obtains the column matrix $[I_o^t]$ associated with the electric current by pre-multiplying the excitation column matrix by the admittance matrix. The elements of the excitation column matrix of (27) of [3] are actually 2π times the slot voltages, but the factor 2π is inconsequential as far as that gain is concerned. DO loop 82 stores $[I_o^t]$ in E3. DO loops 84 and 97 compute the radiation field $E_\theta \sim R_o^{t\theta}[I_o^t]$ and gain G_θ and store them in E and G. The radiation field and gain are normalized so that

$$\int_0^{\pi} G_\theta \sin\theta \, d\theta = 2 \qquad (23)$$

$$|E_\theta|^2 = G_\theta$$

The phase factor $-je^{-jkr}$ is suppressed from E_θ. DO loop 99 prepares horizontal and vertical components E5 and E6 suitable for plotting the gain. DO loop 48 multiplies the eigencurrents by ρ to retrieve the column matrix $[I_o^t]$ associated with each eigencurrent. The index J of DO loop 48 indicates the J^{th} eigencurrent. DO loop 21 stores the matrix $[I] \dfrac{1}{[\tilde{I}][X][I]\ (j + 1/\lambda)}$ appearing in (20) in T3.

The index K of DO loop 29 indicates that inner DO loop 31 will add the contribution from the K^{th} eigencurrent to the admittance matrix (20). DO loop 44 premultiplies $[I_o^t]$ stored in E3 by $\hat{R}_o^{t\theta}$ stored in VR to obtain the radiation field and the gain. The radiation field E_θ and gain G_θ, normalized according to (23), are stored in E and G. In order not to mask a possible discrepancy in the amplitude of the approximate pattern obtained by superimposing a few eigenfields, the present normalization uses the value of the pattern integral (23) previously computed from the admittance matrix obtained by inverting the impedance matrix.

Minimum allocations are given by

```
COMPLEX Z(NM2*NM), Y(NM2*NM), VR(NT*NM), E3(NM),
        T3(NM*NM), E(NT), V(NV)
DIMENSION AMD(JM), RH(NP), ZH(NP), FI(NM2*JM),
        DH(NP-1), TH(NT), T2(NM*NM), X(NM*NM),
        G(NT), L(NC), LV(NV), SN(NT), CS(NT), GX(NT),
        GY(NT), E5(NT*2), E6(NT*2)
COMMON  RS(NP-1), ZS(NP-1), SV(NP-1), CV(NP-1), T(NM*4)
DIMENSION BJ(3*(NP-1))
DIMENSION LR(NM)
```

where

NM2 = NP - 3

NM = NM2/2

Here, NP is its value after execution of statement 90 in the main program. Note that BJ appears in PLANE and LR in LINEQ.

```
//               (0034,EE,3,2,,6),'MAUTZ,JOE',MSGLEVEL=1
// EXEC FORTGCLG,PARM.FORT='MAP'
//FORT.SYSIN DD *
      SUBROUTINE PLANE(VVR,THR,NT)
      COMPLEX VVR(1),A5,A6,U
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,T(80)
      DIMENSION BJ(126),THR(1),FK(20)
      NN=0
      KG=NP-1
      NM=KG/2-1
      M2=NN+2
      A5=2.*3.141593*U**(NN+1)
      FK(1)=1.
      DO 153 J=1,M2
      J1=J+1
      FK(J1)=FK(J)*J
  153 CONTINUE
      DO 156 L=1,NT
      L1=(L-1)*NM
      CS=COS(THR(L))
      SN=SIN(THR(L))
      BCS=BK*CS
      DO 302 J=1,KG
      X=RS(J)*BK*SN
      J1=J
      I1=NN
      IF(I1) 303,304,303
  304 I1=I1+1
      J1=J1+KG
  303 DO 305 JJ=I1,M2
      IF(X-1.E-5) 1,1,2
    1 IF(JJ-1) 3,3,4
    3 BJ(J1)=1.
      GO TO 306
    4 BJ(J1)=0.
      GO TO 306
    2 RH=X/2.
      RH2=RH*RH
      RH3=RH**(JJ-1)
      BJ(J1)=RH3/FK(JJ)
      SS=BJ(J1)
    8 SST=SS*1.E-7
      DO 155 K=1,20
      SS=-SS*RH2/K/(K+JJ-1)
      BJ(J1)=BJ(J1)+SS
      IF(ABS(SS)-SST) 306,306,155
  155 CONTINUE
      STOP 155
  306 J1=J1+KG
  305 CONTINUE
  302 CONTINUE
      IF(NN) 307,308,307
  308 DO 309 J=1,KG
      J1=J+2*KG
      BJ(J)=-BJ(J1)
  309 CONTINUE
  307 DO 300 J=1,NM
      J1=J+L1
      VVR(J1)=0.
      DO 301 I=1,4
```

```
      I1=2*(J-1)+I
      I4=4*(J-1)+I
      I2=I1+KG
      I3=I2+KG
      A6=(COS(ZS(I1)*BCS)+U*SIN(ZS(I1)*BCS))*A5
      BJ2=(BJ(I3)-BJ(I1))*.5
      VVR(J1)=VVR(J1)+A6*(CS*SV(I1)*BJ2+SN*CV(I1)*BJ(I2)*U)*T(I4)
  301 CONTINUE
  300 CONTINUE
  156 CONTINUE
      RETURN
      END
      SUBROUTINE LINEQ(LL,C)
      COMPLEX C(1),STOR,STO,ST,S
      DIMENSION LR(58)
      DO 20 I=1,LL
      LR(I)=I
   20 CONTINUE
      M1=0
      DO 18 M=1,LL
      K=M
      DO 2 I=M,LL
      K1=M1+I
      K2=M1+K
      IF(CABS(C(K1))-CABS(C(K2))) 2,2,6
    6 K=I
    2 CONTINUE
      LS=LR(M)
      LR(M)=LR(K)
      LR(K)=LS
      K2=M1+K
      STOR=C(K2)
      J1=0
      DO 7 J=1,LL
      K1=J1+K
      K2=J1+M
      STO=C(K1)
      C(K1)=C(K2)
      C(K2)=STO/STOR
      J1=J1+LL
    7 CONTINUE
      K1=M1+M
      C(K1)=1./STOR
      DO 11 I=1,LL
      IF(I-M) 12,11,12
   12 K1=M1+I
      ST=C(K1)
      C(K1)=0.
      J1=0
      DO 10 J=1,LL
      K1=J1+I
      K2=J1+M
      C(K1)=C(K1)-C(K2)*ST
      J1=J1+LL
   10 CONTINUE
   11 CONTINUE
      M1=M1+LL
   18 CONTINUE
      J1=0
      DO 9 J=1,LL
```

```
      IF(J-LR(J)) 14,8,14
   14 LRJ=LR(J)
      J2=(LRJ-1)*LL
   21 DO 13 I=1,LL
      K2=J2+I
      K1=J1+I
      S=C(K2)
      C(K2)=C(K1)
      C(K1)=S
   13 CONTINUE
      LR(J)=LR(LRJ)
      LR(LRJ)=LRJ
      IF(J-LR(J)) 14,8,14
    8 J1=J1+LL
    9 CONTINUE
      RETURN
      END
      COMPLEX U,U1,Z(722),Y(1444),VR(1387),E3(19),T3(361),E(73),V(19)
      DIMENSION AMD(38),RH(41),ZH(41),FI(1444),DH(40),TH(73),T2(361)
      DIMENSION X(361),G(73),L(19),LV(19),AREA(400),SN(73),CS(73)
      DIMENSION GX(73),GY(73),XP(2),YP(2),E5(146),E6(146)
      EQUIVALENCE(T2(1),X(1)),(Z(1),Y(1))
      COMMON U,RS(40),ZS(40),SV(40),CV(40),BK,NP,T(80)
      CALL PLOTS(AREA,400)
      READ(1,10) NP,NT,NS,JM,NC,NV,BK,SCL
   10 FORMAT(6I3,2E14.7)
      READ(1,11)(AMD(I),I=1,JM)
   11 FORMAT(5E14.7)
      READ(1,15)(RH(I),I=1,NP)
      READ(1,15)(ZH(I),I=1,NP)
   15 FORMAT(10F8.4)
      READ(1,50)(L(I),I=1,NC)
   50 FORMAT(20I3)
      READ(1,57)(V(I),I=1,NV)
   57 FORMAT(7E11.4)
      READ(1,50)(LV(I),I=1,NV)
      WRITE(3,37)
   37 FORMAT('0 NP NT NS JM NC NV',6X,'BK',12X,'SCL')
      WRITE(3,38) NP,NT,NS,JM,NC,NV,BK,SCL
   38 FORMAT(1X,6I3,2E14.7)
      WRITE(3,5)(AMD(I),I=1,JM)
    5 FORMAT('0AMD'/(1X,5E14.7))
      WRITE(3,39)(RH(I),I=1,NP)
   39 FORMAT('0RH'/(1X,10F8.4))
      WRITE(3,9)(ZH(I),I=1,NP)
    9 FORMAT('0ZH'/(1X,10F8.4))
      WRITE(3,8)(L(I),I=1,NC)
    8 FORMAT('0L'/(1X,20I3))
      WRITE(3,7)(V(I),I=1,NV)
    7 FORMAT('0V'/(1X,7E11.4))
      WRITE(3,6)(LV(I),I=1,NV)
    6 FORMAT('0LV'/(1X,20I3))
      PI=3.141593
      U=(0.,1.)
      IF((RH(1)-RH(NP)).NE.0..OR.(ZH(1)-ZH(NP)).NE.0.) GO TO 90
      RH(NP+1)=RH(2)
      ZH(NP+1)=ZH(2)
      RH(NP+2)=RH(3)
      ZH(NP+2)=ZH(3)
      NP=NP+2
```

```
 90 NM2=NP-3
    NM=NM2/2
    NZ=NM2*NM
    NZ1=NM2*JM
    REWIND 6
    READ(6)
    READ(6)
    READ(6)(Z(I),I=1,NZ)
    READ(6)(FI(I),I=1,NZ1)
    NT2=NT*2
    NT3=NT-NS
    NT4=NS+1
    DO 40 I=2,NP
    I2=I-1
    RR1=RH(I)-RH(I2)
    RR2=ZH(I)-ZH(I2)
    DH(I2)=SQRT(RR1*RR1+RR2*RR2)
    RS(I2)=.5*(RH(I)+RH(I2))
    ZS(I2)=.5*(ZH(I)+ZH(I2))
    SV(I2)=RR1/DH(I2)
    CV(I2)=RR2/DH(I2)
 40 CONTINUE
    DO 41 J=1,NM
    J2=2*(J-1)+1
    J3=J2+1
    J4=J3+1
    J5=J4+1
    J6=4*(J-1)+1
    J7=J6+1
    J8=J7+1
    J9=J8+1
    DEL1=DH(J2)+DH(J3)
    DEL2=DH(J4)+DH(J5)
    T(J6)=DH(J2)*DH(J2)/2./DEL1
    T(J7)=DH(J3)*(DH(J2)+DH(J3)/2.)/DEL1
    T(J8)=DH(J4)*(DH(J5)+DH(J4)/2.)/DEL2
    T(J9)=DH(J5)*DH(J5)/2./DEL2
 41 CONTINUE
    DEL=PI/(NT-1)
    DO 43 J=1,NT
    TH(J)=(J-1)*DEL
    SN(J)=SCL*SIN(TH(J))
    CS(J)=SCL*COS(TH(J))
 43 CONTINUE
    CALL PLANE(VR,TH,NT)
    XP(1)=2.
    XP(2)=8.
    YP(1)=5.
    YP(2)=5.
    DEL1=180./PI
    DO 64 J=1,NT
    TH(J)=TH(J)*DEL1
 64 CONTINUE
    J1=0
    DO 60 J=1,NM
    J2=(J-1)*NM2
    DO 61 I=1,NM
    J1=J1+1
    J3=J2+I
    Z(J1)=Z(J3)
```

```
61 CONTINUE
60 CONTINUE
   J1=0
   DO 71 J=1,NC
   J2=(L(J)-1)*NM2
   DO 72 I=1,NM
   J1=J1+1
   J3=J2+I
   T2(J1)=FI(J3)
72 CONTINUE
   J2=L(J)
   E5(J)=AMD(J2)
71 CONTINUE
   J1=0
   DO 73 J=1,NC
   DO 74 I=1,NM
   J1=J1+1
   FI(J1)=T2(J1)
74 CONTINUE
   AMD(J)=E5(J)
73 CONTINUE
   DO 62 J=1,NM
   J2=(J-1)*NM
   DO 63 I=1,J
   J1=J2+I
   J3=(I-1)*NM+J
   X(J1)=.5*AIMAG(Z(J1)+Z(J3))
   X(J3)=X(J1)
63 CONTINUE
62 CONTINUE
79 CALL LINEQ(NM,Z)
   DO 82 I=1,NM
   E3(I)=0.
   DO 65 K=1,NV
   K1=I+(LV(K)-1)*NM
   E3(I)=E3(I)+Z(K1)*V(K)
65 CONTINUE
82 CONTINUE
   S2=0.
   J2=0
   DO 84 J=1,NT
   U1=0.
   DO 88 I=1,NM
   J2=J2+1
   U1=U1+VR(J2)*E3(I)
88 CONTINUE
   E(J)=U1
   S1=CABS(U1)
   G(J)=S1*S1
   S2=S2+G(J)*SN(J)
84 CONTINUE
   S5=2.*SCL/S2/DFL
   S6=SQRT(S5)
   DO 97 J=1,NT
   E(J)=E(J)*S6
   G(J)=G(J)*S5
97 CONTINUE
   WRITE(3,66)
66 FORMAT('ORADIATION FIELD AND GAIN BY MATRIX INVERSION')
   WRITE(3,67)
```

```
 67 FORMAT('0    0      REAL(E0)     IMAG(E0)      GAIN0')
    WRITE(3,68)
 68 FORMAT('+    -',11X,'-',11X,'-',10X,'-')
    DO 69 J=1,NT,NS
    WRITE(3,70) TH(J),F(J),G(J)
 70 FORMAT(1X,F6.1,3E12.4)
 69 CONTINUE
    DO 99 J=1,NT
    S1=SN(J)*G(J)
    J9=NT2-J+1
    E5(J)=5.+S1
    E5(J9)=5.-S1
    E6(J)=5.+CS(J)*G(J)
    E6(J9)=E6(J)
 99 CONTINUE
    J1=0
    DO 48 J=1,NC
    DO 18 I=1,NM
    J1=J1+1
    J4=2*I+1
    FI(J1)=FI(J1)*RH(J4)
 18 CONTINUE
 48 CONTINUE
    DO 21 J=1,NC
    J1=(J-1)*NM
    S1=0.
    DO 22 I=1,NM
    S2=0.
    J4=(I-1)*NM
    DO 23 K=1,NM
    J2=J4+K
    J3=J1+K
    S2=S2+X(J2)*FI(J3)
 23 CONTINUE
    J3=J1+I
    S1=S1+S2*FI(J3)
 22 CONTINUE
    U1=1./S1/(U+1./AMD(J))
    DO 27 I=1,NM
    J2=J1+I
    T3(J2)=FI(J2)*U1
 27 CONTINUE
 21 CONTINUE
    NZ2=NM*NM
    DO 32 J=1,NZ2
    Y(J)=0.
 32 CONTINUE
    DO 29 K=1,NC
    J3=(K-1)*NM
    J1=0
    DO 31 I=1,NM
    J4=J3+I
    E3(I)=0.
    DO 75 J=1,NV
    J2=J3+LV(J)
    J1=J1+1
    Y(J1)=Y(J1)+T3(J4)*FI(J2)
    E3(I)=E3(I)+Y(J1)*V(J)
 75 CONTINUE
 31 CONTINUE
```

```
      DO 44 J=1,NT,NS
      U1=0.
      J1=(J-1)*NM
      DO 47 I=1,NM
      J2=J1+I
      U1=U1+VR(J2)*E3(I)
   47 CONTINUE
      E(J)=U1*S6
      S1=CABS(E(J))
      G(J)=S1*S1
      GX(J)=5.+SN(J)*G(J)
      GY(J)=5.+CS(J)*G(J)
   44 CONTINUE
      WRITE(3,76) K
   76 FORMAT('0',I3,' MODE RADIATION FIELD AND GAIN')
      WRITE(3,67)
      WRITE(3,68)
      DO 80 J=1,NT,NS
      WRITE(3,70) TH(J),E(J),G(J)
   80 CONTINUE
      CALL LINE(XP,YP,2,1,0,0)
      DO 77 J=1,7
      S1=9-J
      CALL SYMBOL(S1,5.,.14,13,0.,-1)
   77 CONTINUE
      CALL LINE(YP,XP,2,1,0,0)
      DO 78 J=1,7
      S1=9-J
      CALL SYMBOL(5.,S1,.14,13,90.,-1)
   78 CONTINUE
      CALL LINE(E5,E6,NT2,1,0,0)
      DO 86 J=1,NT,NS
      CALL SYMBOL(GX(J),GY(J),.07,4,0.,-1)
   86 CONTINUE
      DO 89 J=NT4,NT3,NS
      J1=NT-J+1
      S1=10.-GX(J1)
      CALL SYMBOL(S1,GY(J1),.07,4,0.,-1)
   89 CONTINUE
      CALL PLOT(7.,0.,-3)
   29 CONTINUE
      CALL PLOT(6.,0.,-3)
      STOP
      END
/*
//GO.FT06F001 DD DSNAME=EE0034.REV1,DISP=OLD,UNIT=2314,                 X
//             VOLUME=SER=SU0004,DCB=(RECFM=V,BLKSIZE=1800,LRECL=1796)
//GO.SYSIN DD *
 21 73  4  5  3  1 0.3141593E+00 0.5000000E+00
 0.3359053E+02 0.6264798E+00-0.1842634E+01-0.1062163E+04-0.1210105E+05
  0.0000  0.5000  1.0000  1.5000  2.0000  2.5000  3.0000  3.5000  4.0000  4.5000
  5.0000  5.5000  6.0000  6.5000  7.0000  7.5000  8.0000  8.5000  9.0000  9.5000
 10.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
  0.0000
  3  4  5
 0.1000E+01 0.0000E+00
  5
/*
```

Output of Program #6

```
NP NT NS JM NC NV        BK                 SCL
21 73  4  5  3  1  0.3141593E 01   0.5000000E 00

AMD
0.3359052E 02 0.6264797E 02-0.1842633E 01-0.1062163E 04-0.1210105E 05

RH
 0.0       0.5000   1.0000   1.5000   2.0000   2.5000   3.0000   3.5000   4.0000   4.5000
 5.0000    5.5000   6.0000   6.5000   7.0000   7.5000   8.0000   8.5000   9.0000   9.5000
10.0000

ZH
 0.0       0.0      0.0      0.0      0.0      0.0      0.0      0.0      0.0      0.0
 0.0       0.0      0.0      0.0      0.0      0.0      0.0      0.0      0.0      0.0
 0.0

L
  3   4   5

V
 0.1000E 01 0.0

LV
  5

RADIATION FIELD AND GAIN BY MATRIX INVERSION

   θ      REAL(Eθ)       IMAG(Eθ)       GAINθ
  0.0   0.0            0.0            0.0
 10.0  -0.5429E 00     0.3129E 00     0.3926E 00
 20.0  -0.9745E 00     0.5559E 00     0.1259E 01
 30.0  -0.1223E 01     0.6866E 00     0.1967E 01
 40.0  -0.1273E 01     0.7001E 00     0.2111E 01
 50.0  -0.1157E 01     0.5215E 00     0.1725E 01
 60.0  -0.9292E 00     0.4472E 00     0.1101E 01
 70.0  -0.6397E 00     0.3285E 00     0.5171E 00
 80.0  -0.3239E 00     0.1640E 00     0.1318E 00
 90.0  -0.5844E-06     0.2944E-06     0.4281E-12
100.0   0.3239E 00    -0.1640E 00     0.1318E 00
110.0   0.6397E 00    -0.3285E 00     0.5171E 00
120.0   0.9292E 00    -0.4472E 00     0.1101E 01
130.0   0.1157E 01    -0.5215E 00     0.1726E 01
140.0   0.1273E 01    -0.7001E 00     0.2111E 01
150.0   0.1223E 01    -0.6866E 00     0.1967E 01
160.0   0.9745E 00    -0.5559E 00     0.1259E 01
170.0   0.5429E 00    -0.3129E 00     0.3926E 00
180.0   0.0            0.0            0.0
```

1 MODE RADIATION FIELD AND GAIN

θ	REAL(Eθ)	IMAG(Eθ)	GAINθ
0.0	0.0	0.0	0.0
10.0	-0.5762E 00	0.3127E 00	0.4299E 00
20.0	-0.1024E 01	0.5557E 00	0.1357E 01
30.0	-0.1265E 01	0.6863E 00	0.2070E 01
40.0	-0.1289E 01	0.6998E 00	0.2152E 01
50.0	-0.1145E 01	0.6212E 00	0.1696E 01
60.0	-0.8974E 00	0.4870E 00	0.1042E 01
70.0	-0.6051E 00	0.3284E 00	0.4739E 00
80.0	-0.3020E 00	0.1639E 00	0.1181E 00
90.0	-0.5422E-06	0.2943E-06	0.3806E-12
100.0	0.3020E 00	-0.1639E 00	0.1181E 00
110.0	0.6051E 00	-0.3284E 00	0.4739E 00
120.0	0.8974E 00	-0.4870E 00	0.1042E 01
130.0	0.1145E 01	-0.6212E 00	0.1696E 01
140.0	0.1289E 01	-0.6998E 00	0.2152E 01
150.0	0.1265E 01	-0.6863E 00	0.2070E 01
160.0	0.1024E 01	-0.5557E 00	0.1357E 01
170.0	0.5762E 00	-0.3127E 00	0.4299E 00
180.0	0.0	0.0	0.0

2 MODE RADIATION FIELD AND GAIN

θ	REAL(Eθ)	IMAG(Eθ)	GAINθ
0.0	0.0	0.0	0.0
10.0	-0.5422E 00	0.3127E 00	0.3918E 00
20.0	-0.9735E 00	0.5557E 00	0.1256E 01
30.0	-0.1222E 01	0.6862E 00	0.1964E 01
40.0	-0.1273E 01	0.6997E 00	0.2109E 01
50.0	-0.1157E 01	0.6212E 00	0.1725E 01
60.0	-0.9296E 00	0.4870E 00	0.1101E 01
70.0	-0.6402E 00	0.3284E 00	0.5178E 00
80.0	-0.3243E 00	0.1639E 00	0.1320E 00
90.0	-0.5852E-06	0.2943E-06	0.4290E-12
100.0	0.3243E 00	-0.1639E 00	0.1320E 00
110.0	0.6402E 00	-0.3284E 00	0.5178E 00
120.0	0.9296E 00	-0.4870E 00	0.1101E 01
130.0	0.1157E 01	-0.6212E 00	0.1725E 01
140.0	0.1273E 01	-0.6997E 00	0.2109E 01
150.0	0.1222E 01	-0.6862E 00	0.1964E 01
160.0	0.9735E 00	-0.5557E 00	0.1256E 01
170.0	0.5422E 00	-0.3127E 00	0.3918E 00
180.0	0.0	0.0	0.0

3 MODE RADIATION FIELD AND GAIN

θ	REAL(Eθ)	IMAG(Eθ)	GAINθ
0.0	0.0	0.0	0.0
10.0	-0.5416E 00	0.3127E 00	0.3912E 00
20.0	-0.9726E 00	0.5557E 00	0.1255E 01
30.0	-0.1221E 01	0.6862E 00	0.1962E 01
40.0	-0.1272E 01	0.6997E 00	0.2109E 01
50.0	-0.1158E 01	0.6212E 00	0.1726E 01
60.0	-0.9301E 00	0.4870E 00	0.1102E 01
70.0	-0.6407E 00	0.3284E 00	0.5183E 00
80.0	-0.3245E 00	0.1639E 00	0.1322E 00
90.0	-0.5857E-06	0.2943E-06	0.4296E-12
100.0	0.3245E 00	-0.1639E 00	0.1322E 00
110.0	0.6407E 00	-0.3284E 00	0.5183E 00
120.0	0.9301E 00	-0.4870E 00	0.1102E 01
130.0	0.1158E 01	-0.6212E 00	0.1726E 01
140.0	0.1272E 01	-0.6997E 00	0.2109E 01
150.0	0.1221E 01	-0.6862E 00	0.1962E 01
160.0	0.9726E 00	-0.5557E 00	0.1255E 01
170.0	0.5417E 00	-0.3127E 00	0.3912E 00
180.0	0.0	0.0	0.0

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13. ABSTRACT

Computer programs are given for calculating the characteristic currents and characteristic gain patterns of conducting bodies of revolution. Also given are computer programs for using these characteristic currents in aperture radiation and plane-wave scattering problems. Plot programs for use with a Calcomp plotter are included. Operating procedures and program details are discussed, and sample input-output data are given.

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